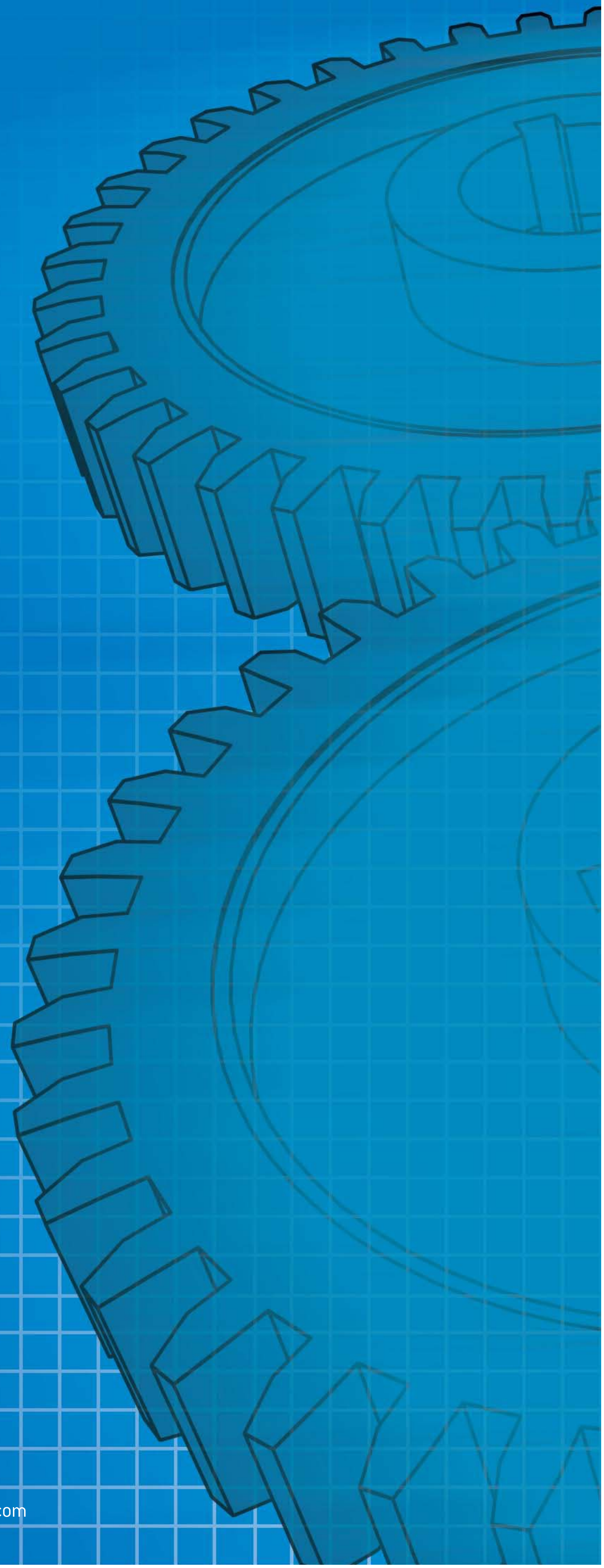
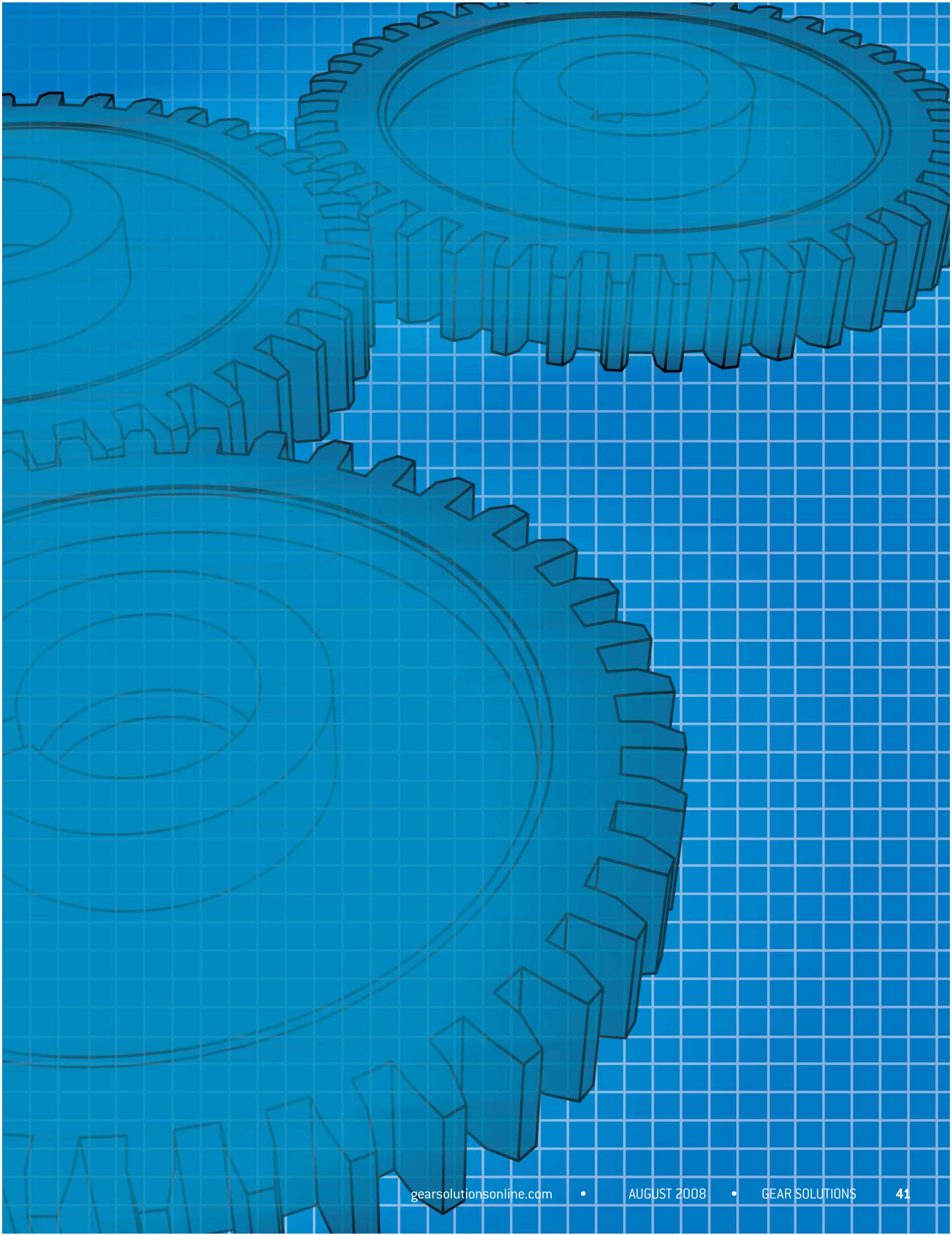
AN ADVANCED Approach to OPTIMAL Gear Design

By Veniamin I. Goldfarb, Alexander L. Kapelevich, and Alexander A. Tkachev

Take two proven design approaches—Direct Gear Design and the Method of Dynamic Blocking Contours—combine their best attributes, and the result is Advanced Gear Design. Read on to learn more.





The theory and practice of gear design is rich with examples of successful and sometimes outstanding combinations of various mathematical, manufacturing, and methodological approaches to the choice of the best parameters, defining the gear itself, its generating tools, and processes of generation and meshing of tooth flanks [1, 2, 3, and others]. Knowledge of involute spur and helical gears can be considered to be almost complete compared with other types of gears, and they also apply different approaches to the optimal (or rational) choice of their geometrical parameters.

This paper proposes a new approach—Advanced Gear Design (AGD)—on the basis of two reputed approaches, one of which is known as Direct Gear Design® (DGD) [4, 5, 6] and the other can be named as the Method of Dynamic Blocking Contours (DBC) [7, 8, 9]. The proposed approach uses the advantages of the first two in order to obtain the best decision when designing involute spur and helical gears.

1: What We Imply By Optimal Design

We are going to consider two tasks that are typical for the optimal design of any product, including a gear:

- Optimizing the design, implying the choice of the best design decision according to the given criterion or a group of criteria;
- Optimizing the process of design, implying the increase of its productivity and quality.

The proposed concept of the first task solution means the rejection of the traditional approach to the design of involute spur and helical gears, where the first step is the assignment of parameters (as a rule, standard ones) of the initial generating tool (generating rack). Such an approach is justified in many practical cases from the manufacturing point of view, but it has restrictions, imposed both on the number of gear parameters and on the choice of the optimal decision. In the proposed approach the tool parameters are secondary and are determined after the gear parameters are chosen according to the given quality criteria with account of possible restrictions of design (geometrical) and manufacturing character. Here, any alterations of the tooth shape—including asymmetry—are possible, which provide necessary gear performance [4].

The basis of the approach to the second task solution is such a design version, when part of the parameters—let's call them key ones—which greatly influence the gear performance, are chosen at the early stage of the process, affiliating the assignment of initial data, on the basis of the results of previous calculations and investigations, accumulated in the form of tables, graphs, formulas, or anything else. The experience of practical implementation of such an approach shows that the dual effect of optimization can be achieved here:

- Increase of the design process productivity due to the fact that a number of gear performance characteristics can be forecast (predicted) at the early stage of this process, omitting complex and lengthy calculations;
- Achieving the best gear quality according to those characteristics, which are possible to forecast with the help of the mentioned key parameters.

2. Direct Gear Design Approach

The idea of Direct Gear Design is not new. Ancient engineers successfully used it centuries ago, using the desired gear performance and known operating conditions to define gear geometry. Then they made gear drives according to this geometry using available materials, technology, and tools.

It is important to note that the gear geometry was defined first. In other words, gear parameters were primary, and the manufacturing process and tool parameters were secondary. This is an essence of Direct Gear Design. This design approach is developed for involute gears and based on the Theory of Generalized Parameters created by Prof. E.B. Vulgakov [4], and it can be defined as an application-driven gear drive development process with primary emphasis on performance maximization and cost efficiency without concern for any predefined tooling parameters.

2.1. Gear Tooth And Mesh Synthesis

There is no need for a basic (or generating) gear rack to describe the gear tooth profile. Two involutes of the base circle, the arc distance between them, and tooth tip circle describe the gear tooth (Fig.1). The equally spaced teeth form the gear. The fillet between teeth is not in contact with the mating gear teeth. However, this portion of the tooth profile is critical because this is the area of the maximum bending stress concentration.

Two (or more) gears with the equal base circle pitch can be put in mesh (Fig.2). The operating pressure angle αw and the contact ratio $\varepsilon\alpha$ for the gear with symmetric teeth are defined by the following formulae [4, 5]:

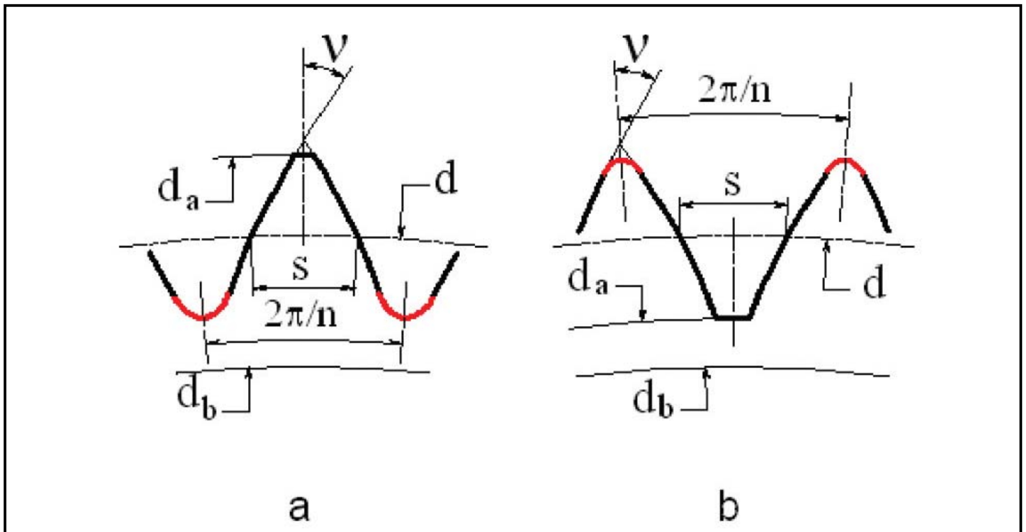


Fig. 1: Tooth profile (the fillet portion is red); a, external geartooth; b, internal gear tooth; d_a, tooth tip circle diameter; d_b, base circle diameter; d, reference circle diameter; S, circular tooth thickness at the reference diameter; ν, involute intersection profile angle.

For external gearing:

α_w= arcinv [(inv ν_1+ u * inv ν_2 − π / n_1) / (1 + u)],

ε_α = n1 * [tan α_{a1} + u * tan α_{a2}–(+ u) * $\tan_w$] / (2 *π)

For internal gearing:

α_w = arcinv [(u * inv ν_2 – inv ν_1) / (u – 1)],

ε_α = n_1 * [tan α_{a1}–u * tan α_{a2} +(u – 1) * tan α_w] / (2 *π).

Where n_1 and n_2 are pinion and gearwheel numbers of teeth

u = n_2 / n_1 is the gear ratio;

α_a = arcos (d_b/d_a) is the involute profile angle at the tooth tip diameter.

For metric system gears the operating module is m_w = 2*a_w/(n_2±n_1). For English system gears the operating diametral pitch is p_w=(n_2 ± n_1)/(2*a_w). The "+" is for the external gearing and the "-" is for the internal gearing.

2.2. Tooth Fillet Profile Design and Optimization

In traditional gear design the fillet profile is a trajectory of the tool cutting edges in generating motion. The most common way to reduce bending stress concentration is using the full radius generating rack. In some cases the generating rack tip as formed by parabola, ellipsis, or other mathematical curves. All these approaches have limited effect on bending stress reduction, which depends on the generating rack profile angle and number of gear teeth.

In Direct Gear Design the fillet profile is optimized in order to minimize bending stress concentration. The initial fillet profile is a trajectory of the mating gear tooth tip in the tight (zero backlash) mesh. The FEA and random search method are used for fillet optimization [6]. The approximate center of the initial fillet is connected with the fillet finite element nodes. During the optimization process the random search method is moving the fillet nodes (except first and last) along the beams (see Fig. 3a). The bending stresses are calculated for every new fillet point combination. If the maximum bending stress is reduced, the program continues the search in the same direction. If not, it steps back and starts searching the different direction. After a certain number of iterations the calculation process results with forming the optimized fillet profile that provides minimum achievable bending stress (Fig. 3b).

This fillet provides the minimized radial clearance with the mating gear tooth, excluding interference at the worst tolerance combination and operating conditions. It also has the maximized curvature radius, distributing the bending stress along a large portion of the fillet, reducing stress concentration (Fig. 4). The shape of the optimized fillet profile depends on the mating gear geometry. However, it practically does not depend on the load level and load application point.

The Table 1 presents bending stress reduction, achievable by the full radius rack application and by the fillet profile optimization in comparison to the standard 20˚ and 25˚ rack for gears with different number of teeth. The involute portion of the tooth profile is the same.

2.3. Bending Stress Balance

Mating gears should be equally strong. If the initially calculated bending stresses for the pinion and the gear are significantly different, the bending stresses should be balanced [6].

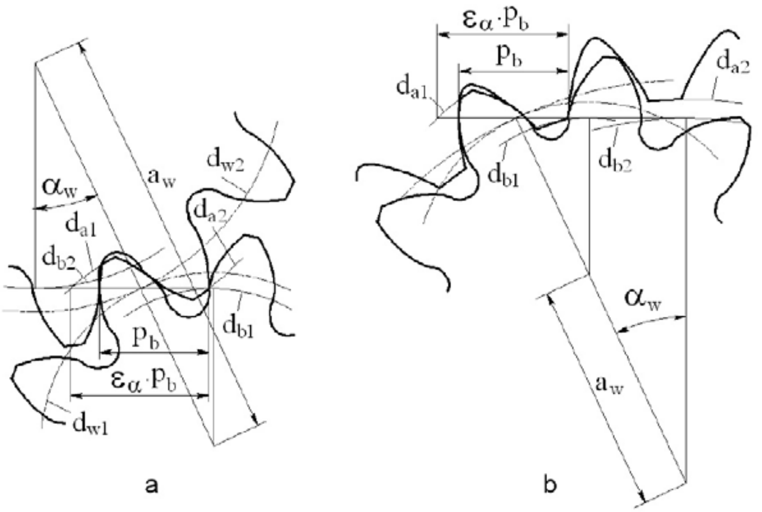


Fig. 2: Gear mesh; a, external gearing; b, internal gearing; a_w, center distance; p_b, base circle pitch, α_w - operating pressure angle; ε_α, contact ratio; $d_{w1,2}$, operating pitch circle diameters; subscripts "1" and "2" are for the mating pinion and the gear.

DGD defines the optimum tooth thickness ratio S_{w1}/S_{w2} (Fig. 5), using FEA and an iterative method, providing a bending stress difference of less than 1 percent. If the gears are made out of different materials, the bending safety factors should be balanced.

Direct Gear Design is applicable for all kinds of involute gears: the spur gears including external, rack, and pinion, and external, helical, bevel, worm, and face gears, etc. The helical, bevel, and worm gear tooth profile is typically optimized in the normal section. The face gear fillet is different in every section along the tooth line. Therefore its profile is optimized in several sections and then is blended into the fillet surface.

So, the Direct Gear Design method presented here provides complete gear tooth profile optimization, resulting in significant contact and bending stress reduction. This stress reduction is converted to:

- Higher load capacity
- Reduced size and weight
- Extended lifetime
- Reduced noise and vibration
- Higher efficiency
- Higher reliability
- Reduced cost

3. Method of Dynamic Blocking Contours

This method is applied at the initial stage of gear design, which is related to the definition of shift coefficients of the pinion (x_1) and gear-

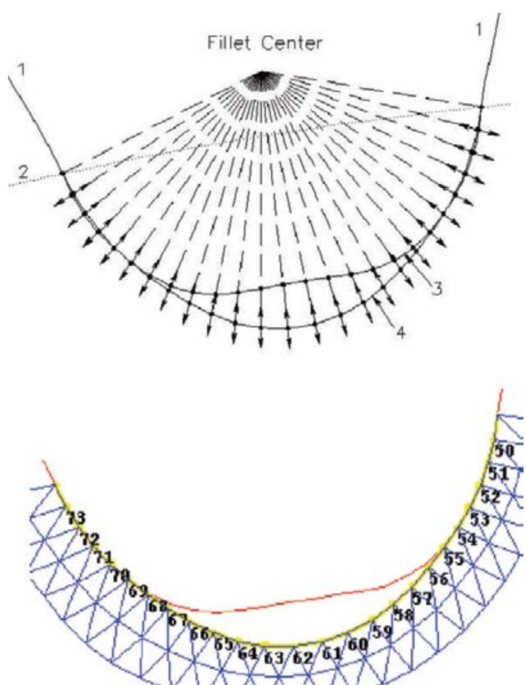


Fig. 3: The fillet profile optimization; a, the random search method application, b, the FEA mesh around the optimized fillet.

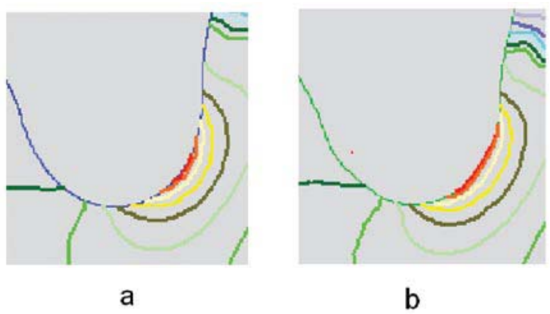


Fig. 4: Bending stress distribution along the fillet; a, the full radius rack generated fillet; b, the same tooth with the optimized fillet.

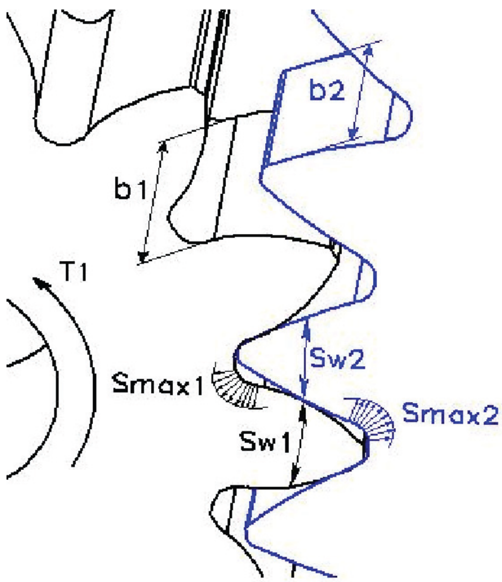


Fig. 5: Balance of the maximum bending stresses

Pinion and gear number of teeth	Pinion and gear number of teeth Bending stress reduction in comparison with the standard 20° rack, %		Bending stress reduction in comparison with the standard 25° rack, %	
	Full radius 20° rack	Optimized fillet	Full radius 25° rack	Optimized fillet
12	-	-	8	21
15	6	25	7	20
20	10	23	6	18
30	10	21	6	17
50	10	21	5	15
80	10	21	5	14
120	10	21	4	13

Table 1:

wheel (x_2), which determine the tooth shape and many characteristics of gear quality, to a great extent.

For about 50 years the approach [7] has been known to define rational values of these coefficients, based on application of blocking contours; geometrical objects in the coordinate plane x_1, x_2, limited by an assemblage of lines which display graphically main restrictions which are necessary to meet for cinematically correct gear operation—absence of tooth undercut, sharpening and interference, providing minimum allowable transverse contact ratio $\varepsilon_\alpha = 1$, and so on. Intersecting lines, corresponding to the mentioned restrictions, make up a closed area in the plane x_1, x_2, and within its area a point (x_1, x_2) must be found which determines allowable values of pinion and gearwheel shift coefficients. A closed curve, limiting this area, is called a blocking contour (BC).

Fig.6 shows the example of a typical BC, constructed for a spur gear with tooth numbers of the pinion $n_1 = 20$ and gearwheel $n_2 = 45$ and standard initial contour of the generating rack (profile angle $\alpha = 20°$, addendum coefficient $h_a^* = 1$, radial clearance coefficient $c^* = 0{,}25$).

The possibilities of BC are not exhausted by given lines. Several additional lines can be displayed in the plane x_1, x_2 (Fig.7), reflecting a number of gear quality characteristics [8, 9]: A is the line of given center distance α_w (line of constant coefficient of shift sum $x_\Sigma = x_1 + x_2$); B is the line of increased contact strength (line of maximum transverse contact ratio); and C is the line of equal specific sliding (line of increased wear resistance). The construction of other lines is possible; for example, a line of maximum/minimum value $a_{w\ max/min}$ for given initial data. The choice of the point (x_1, x_2) on one of these additional lines means achievement of the corresponding gear quality or parameter.

Using the concept of BC allows one to assign (forecast) the definite quality of a gear at the early stage of its design, omitting complex and time-consuming calculations and providing the increase of design productivity—moreover, including the optimization problem.

In order to implement this method widely, based on the application of blocking contours, into the practice of spur and helical gear design, a great number of blocking contours [10] has been calculated and published, mainly in Russian technical literature. However, application of such "classic" BC is connected with essential restrictions, since they were calculated and constructed for:

- Spur gears (when the tooth helix angle is $\beta = 0°$; for helical gears when $\beta \neq 0°$, equivalent tooth numbers should be used $n_{equiv} = n/\cos^3\beta$; the accuracy of defining shift coefficients is decreased here);
- Fixed combinations of tooth numbers n_1 and n_2;
- Standard values of initial contour parameters ($\alpha = 20°$, $h_a^* = 1$, $c^* = 0{,}25$).

The pointed restrictions, which are not crucial in the modern practice of gear design, limit the application of the concept of BC for more flexible design methods used nowadays and, in particular; contradict the concept of Direct Gear Design. In this case, a traditional approach can be changed by the method of dynamic blocking contours (DBC) [8, 9], which is the evolution of the concept of BC and is implemented in the computer system "contour."

The essence of the DBC method is a special approach to the calculation and display of those lines of BC, whose configuration depends not only on the parameters of a gear itself (n_1, n_2, α, h_a^*, c^*, β), but on the values of corresponding quality characteristic. For example, on transverse contact ratio ε_α or coefficient $k_{1(2)}$ of thickness ($S_{a1(2)}$) of pinion (gearwheel) tooth at the addendum circle ($S_{a1,2} = k_{1,2}\, m$, m is the module). When assigning the range or discrete values ε_α (or $k_{1,2}$), two or more lines of the corresponding quality characteristic can be simultaneously found in the plane x_1, x_2. Here, the user can forecast with confidence that choosing the point (x_1, x_2) within the area closed between two lines—for example, $\varepsilon_\alpha = 1{,}1$ and $\varepsilon_\alpha = 1{,}5$ (Fig.7)—he will obtain the value ε_α within the range $1{,}1 \leq \varepsilon_\alpha \leq 1{,}5$. Along with the possibility to

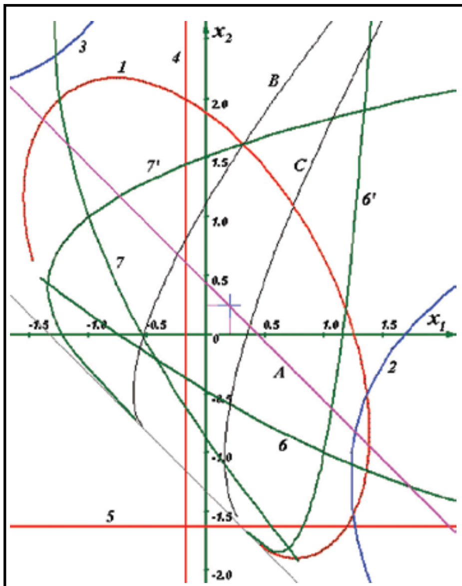


Fig. 6: Blocking contour of a gear. 1, line of transverse contact ratio ($\varepsilon_\alpha = 1$), 2 and 3, lines of tooth sharpening of the pinion ($S_{a1} = 0$) and gearwheel ($S_{a2} = 0$) correspondingly, 4 and 5, lines of tooth undercut of the pinion and gearwheel, 6 and 6', lines of interference of the pinion (when the gearwheel tooth apex tends to interfere the pinion tooth root), 7 and 7', lines of interference of the gearwheel (the reverse situation).

alter interactively any pointed gear parameter, the user can dynamically influence the variation of lines configuration, which form the BC itself (this explains the origin of the name "method of dynamic BC").

One should note that the practical implementation of the concept of DBC, and the appearance of the concept itself, became possible only within the development of computer-aided gear design on the basis of modern computer techniques, when the time-consuming calculation procedure of BC lines stopped to be the limiting factor.

The concept of DBC can be spread to some other lines to be dealt with when applying blocking contours. They are, in particular, lines defining areas in the plane x_1, x_2 where the pitch point can be displaced to the zone of two-pair meshing. Here, the definition feature is the value δ, which is equal (in module parts) to the distance between the pitch point and the boundary of zones of one- and two-pair meshing, and it shows how far the pitch point is in the zone of two-pair meshing.

The DBC method allows one to complicate and considerably vary the tasks that are traditionally solved with the help of blocking contours. Here are some examples:

- When the center distance a_w is given, shift coefficients must be determined which provide: a) maximum smoothness of gear operation; b) increased resistance to scuffing and abrasive wear; c) reasonable compromise between these two versions;
- Shift coefficients must be determined, which provide increased contact strength of a gear;
- When initial parameters are given, minimum and maximum center distances and corresponding shift coefficients must be determined.

Therefore, application of DBC method increases considerably possibilities of optimizing the design of spur and helical gears with relation both to the choice of optimal parameters according to the given criterion or a group of criteria, and to the increase of design process productivity.

4. Combined Method

The tendency to enhance gears and the methods of their design leads to the idea of the rational compromise of both the approaches considered above, which can supply gear designers with an effective gear development tool. Let's consider the possibilities of generating some combined approach—Advanced Gear Design (AGD):

- A gear must be reversible and provide equal loading conditions when operating with right and left tooth flanks; this requirement normally determines the symmetry of gearwheel teeth;
- A gear transmits rotation (and load) in one direction and, therefore, only one of tooth flanks is loaded; in this case any alteration of tooth shape (including asymmetry) is possible, providing better quality of a gear under given conditions.

In the first case for the gear design with the symmetric teeth, if the DGD is used, it completely defines tooth geometry of the mating gears. Although this is the optimal solution for the required gear performance, it typically does not allow using the common generating (tooling) rack, which might be preferable for gear fabrication. If the DBC method is used and the numbers of teeth, module, helix angle (for helical gears), and the generating rack parameters are selected, this allows constructing the blocking contour and chose the shift coefficients x_1 and x_2, defining the gear geometry. This approach is certainly more manufacturing friendly than the DGD, although it compromises some gear performance (for example, bending stress reduction).

In the second case for the gear design with the asymmetric teeth, the DGD defines the optimized gear geometry the similar way only using the different base circle for each tooth flank. The DBC is also similar to the first case with the only following difference: each tooth

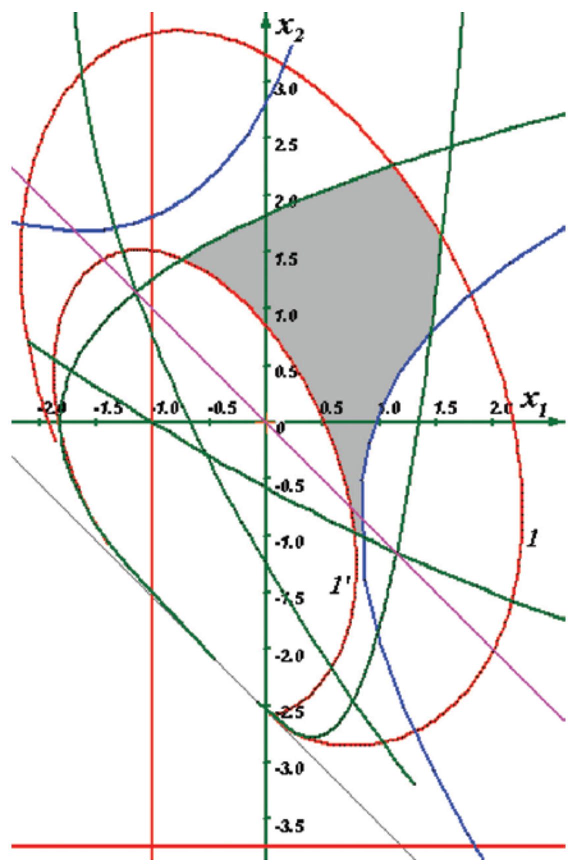


Fig. 7: Blocking contour with restriction $1{,}1 \le \varepsilon_\alpha \le 1{,}5$: line 1 – ε_α = 1,1; 1′ – ε_α = 1,5 (n_1 = 22; n_2 = 50; α = 25°; h_a^* =1,15; β = 15°).

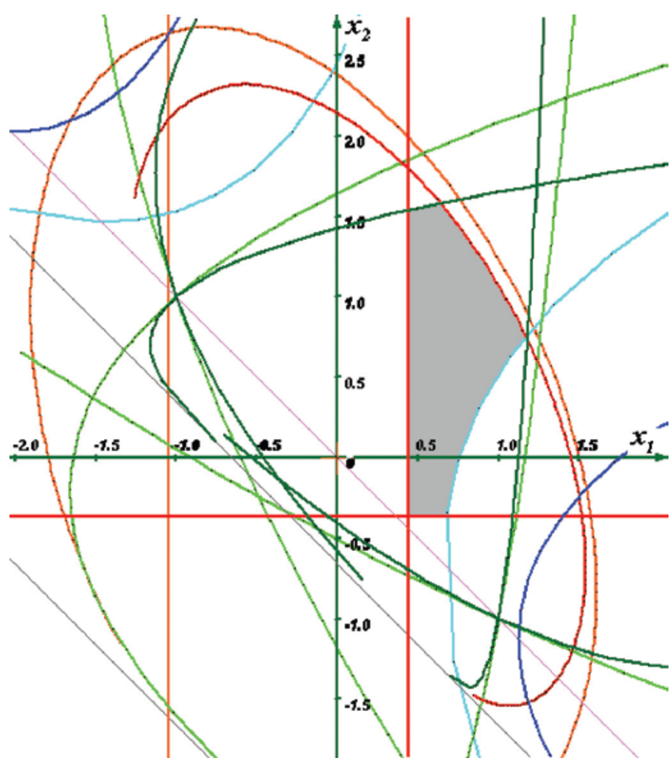


Fig. 8: Superimposed blocking contour: the area of allowable values of shift coefficients, common to both BC, is shadowed (n_1 = 15; n_2 = 37; α_1 = 15°; α_2= 30°; h_a^* =1; β = 15°. The α_1 and α_2 are the profile angles of the asymmetric generating rack).

side will have its own blocking contour. Here, the shift coefficient can be either the same for both tooth sides (if the generation of these sides is performed simultaneously and by a single tool), or it can be chosen separately for each side, if they are generated separately and by different tools. In the first of these cases, the choice can be performed by means of "superimposed" blocking contour (Fig. 8) obtained by imposing the BC of the left tooth flank on the BC of the right tooth flank, in the second case, by means of two separate BC.

According to the concept of DGD, for both versions the sequence of design implies first the definition of gear parameters with maximum account of imposed requirements and then the solution of gear manufacturing problems and the definition of tool geometry. The latter manufacturing tasks are independent and are not considered here.

It is evident that the design according to the AGD method is performed automatically—that is, with the help of a computer. Similar to uniting the design methods, their implementing program systems can be integrated here, with each of them becoming the module (subsystem) of the integrated CAD-system.

Conclusion

This paper describes a new approach to the process of involute spur and helical gear design, assembling the advantages of two new methods, which occurred not long ago and are now being developed intensively—that is, the method of Direct Gear Design (DGD) and of Dynamic Blocking Contours (DBC). The proposed combined method, called Advanced Gear Design (AGD), logically unites the advantages of these methods. Practical implementation of the new method allows: 1) to look at the process of computer-aided design of involute spur and helical gears in a new way, enriching considerably its contents and results; 2) to create on its base new generation gears with considerably better characteristics and with the possibility of their application in new mechanisms and machines.

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ABOUT THE AUTHORS:

Dr. Veniamin I. Goldfarb is a professor in the production mechanical engineering department of Izhevsk State Technical University, located in Izhevsk, Russia. He's also the department's head and is director of the university's Institute of Mechanics. He specializes in theory and practice of gear design, investigation, and production, and he is also chair of the gearing technical committee of the International Federation for the Theory of Machines and Mechanisms. Dr. Alexander L. Kapelevich is an owner and president of consulting firm AKGears, LLC, located in Shoreview, Minnesota. He's a developer of the modern Direct Gear Design® approach and an active member of the AGMA's Aerospace and Plastic Gearing Committees. Dr. Alexander A. Tkachev is an assistant professor in the production mechanical engineering department of Izhevsk State Technical University, where he's mainly responsible for programming gear design software, especially for designing spur and helical gears. He is also an executive engineer of Izhevsk Mechanical Plant, a company that produces power tolls and other products. As an executive engineer, he is involved in developing CAD systems for gears. The university's Web site is [inter.istu.ru]. Visit AK Gears at [www.akgears.com].