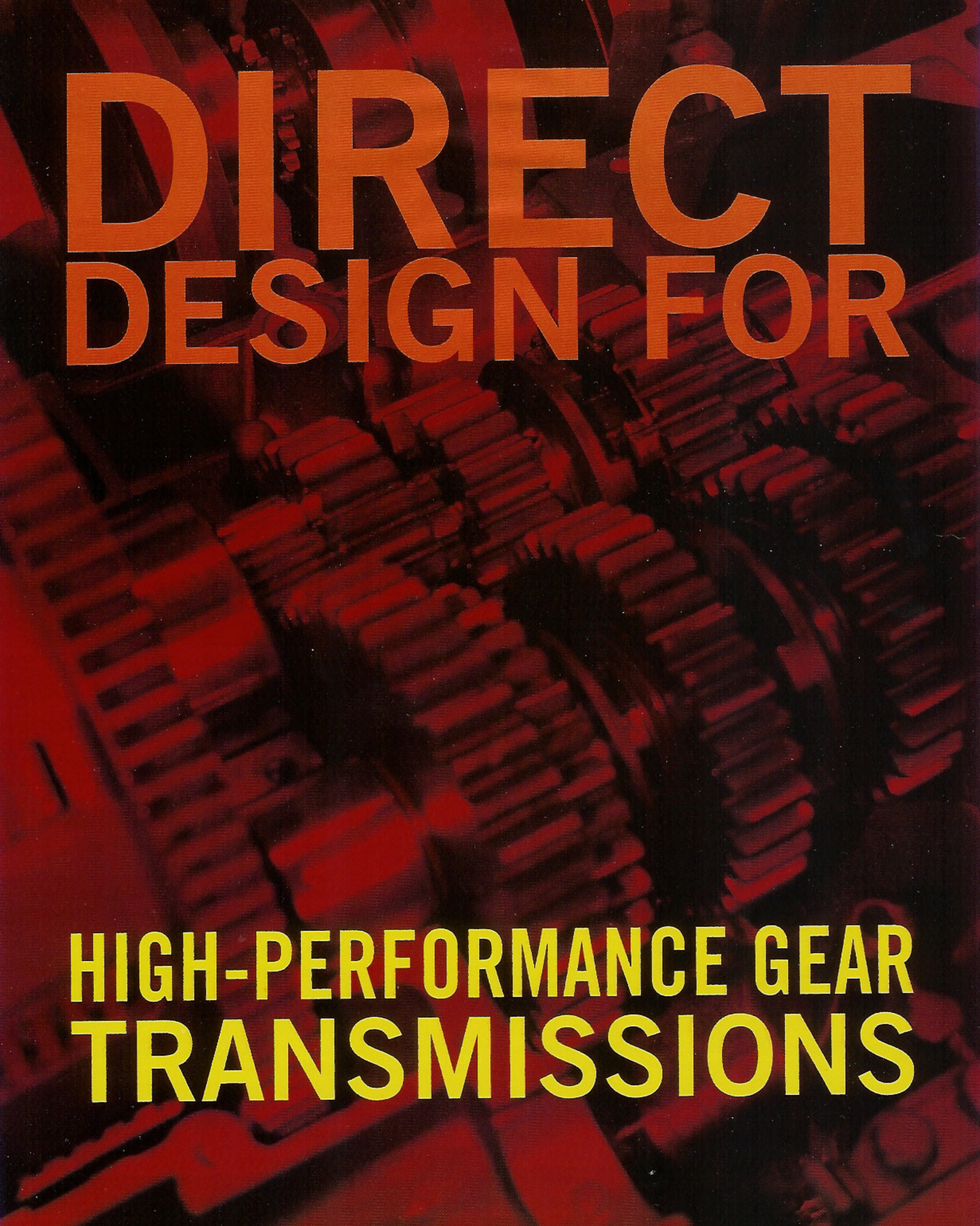

DIRECT DESIGN FOR HIGH-PERFORMANCE GEAR TRANSMISSIONS

FOR DEMANDING APPLICATIONS WHERE CUSTOM GEARING IS REQUIRED, THE DIRECT GEAR DESIGN METHOD MAY BE JUST THE APPROACH YOU'RE LOOKING FOR. THIS ARTICLE PRESENTS AN IN-DEPTH EXPLANATION OF THIS EXCITING PROCESS.

By Alexander Kapelevich

This paper presents a unique methodology for designing gears to enhance strength and life while allowing size and wear reduction. This new approach, trade named Direct Gear Design (DGD), optimizes the gear geometry to impart superior drive performance versus traditional gear design methods. This paper explains this alternative approach and demonstrates its effectiveness for high performance gear transmissions.

Introduction

Traditional standard gear design [1] is based on basic (or generating) gear rack. Parameters of this basic gear rack such as a profile angle, addendum, whole depth, and fillet radius proportions were standardized and used as input gear design parameters. The 20° pressure angle or other basic gear racks became a starting point and foundation of gear design. This made gear design indirect depending on pre-selected, typically standard, basic gear rack parameters. Major benefits provided by traditional gear design approach are gear interchangeability and low tooling inventory. For better gear performance (higher efficiency, load capacity, etc.) there are addendum modifications (or X-shifts) recommended by standards. However, selection of the standard basic rack imposes certain limitations on gear performance because, in this case, all possible gear solutions are inside the so-called block-contours [2]. In order to improve gear performance beyond the block-contour limit, the non-standard basic rack or individual basic racks for the mating gears should be considered. Besides, there is a clear trend to gear customization and interchangeability of gears is not very critical anymore. Low tooling inventory is not that important, either. Leading gear applications (aerospace and automotive, for example) do not use the standard basic racks for gear design, replacing them with the custom racks (with increased pressure or contact ratio, for example), providing higher gear performance.

The gear generating fabrication process (or hobbing), which uses the generating rack as a cutting edge of the tool, is not so dominating any more in gear production. Gears are also made by profile cutting, grinding, broaching, etc. There are many gear forming technologies, such as precision forging, casting, and extrusion, powder metal processing, plastic and metal injection molding, etc. All these technologies do not use the gear generating method. However, they are used to make gears, traditionally designed based on pre-selected gear racks. The proposed alternative Direct Gear Design method does not use the basic rack parameters. It uses desired performance parameters and operating conditions to define gear shape.

The idea of Direct Gear Design is not new. Ancient engineers successfully used it centuries ago. They used desired gear performance and known operating conditions to define gear geometry. Then they made gear drives to this geometry using available materials, technology and tools. It is important to note that the gear geometry was defined first. In other words gear parameters were primary, and the manufacturing process and tool parameters were secondary. This is the essence of Direct Gear Design.

This design approach is developed for involute gears and based on the Theory of Generalized Parameters created by Prof. E.B. Vulgakov [3] and can be defined as an application driven gear drive development process with primary emphasis on performance maximization and cost efficiency without concern for any predefined tooling parameters.

Gear Tooth and Mesh Synthesis

There is no need for a basic (or generating) gear rack to describe the gear tooth profile. Two involutes of the base circle, the arc distance between them, and tooth tip circle describe the gear tooth (Fig.1). The equally spaced teeth form the gear. The fillet between teeth is not in contact with the mating gear teeth. However, this portion of the tooth profile is critical because this is the area of the maximum bending stress concentration.

Two (or more) gears with the equal base circle pitch can be put in mesh (Fig.2). The operating pressure angle α_w and the contact ratio ε_α are

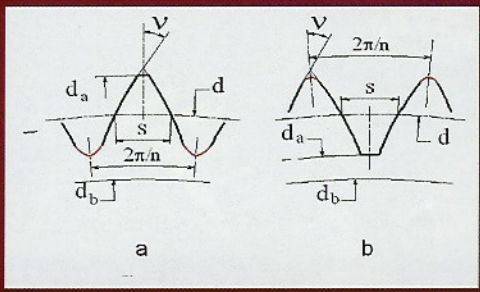

FIG 1: TOOTH PROFILE (THE FILLET PORTION IS RED); A—EXTERNAL TOOTH; B—INTERNAL TOOTH; d_a—TOOTH TIP CIRCLE DIAMETER; d_b—BASE CIRCLE DIAMETER; D—REFERENCE CIRCLE DIAMETER; S—CIRCULAR TOOTH THICKNESS AT THE REFERENCE DIAMETER; ν—INVOLUTE INTERSECTION PROFILE ANGLE.

defined by the following formulae [3, 4]:

For external gearing

α_w = arcinv [(inv ν_1+ u ∗ inv ν_2 − π/n_1) / (1 + u)],

$\varepsilon\alpha$ = n_1 ∗ [tan α_{a1}+ u ∗ tan α_{a2} (− 1 + u) ∗ tan α_w]/ (2∗π).

For internal gearing

α_w = arcinv [(u ∗ inv ν_2 − inv ν_1) / (u − 1)],

ε_α = n_1 ∗ [tan α_{a1}−u ∗ tan α_{a2}+ (u − 1) ∗ tan α_w] / (2 ∗ π).

Where

u = n_2 / n_1 is the gear ratio;

α_a = arcos (d_b/d_a) is the involute profile angle at the tooth tip diameter.

For metric system gears the operating module is m_w = 2 ∗ a_w / (n2 ± n1). For English system gears the operating diametral pitch is p_w = (n2 ± n1) / (2 ∗ a_w). The "+" is for the external gearing and the "-" is for the internal gearing.

Tooth Fillet Profile Design and Optimization

In order to complete the nominal gear geometry description, the tooth fillet profile should be defined. In traditional gear design the fillet profile is a trajectory of the tool cutting edges in generating motion. The most common way to reduce bending stress concentration is using the full radius generating rack. In some cases the generating rack tip as formed by parabola, ellipsis, or other mathematical curves. All these approaches have limited effect on bending stress reduction, which depends on the generating rack profile angle and number of gear teeth.

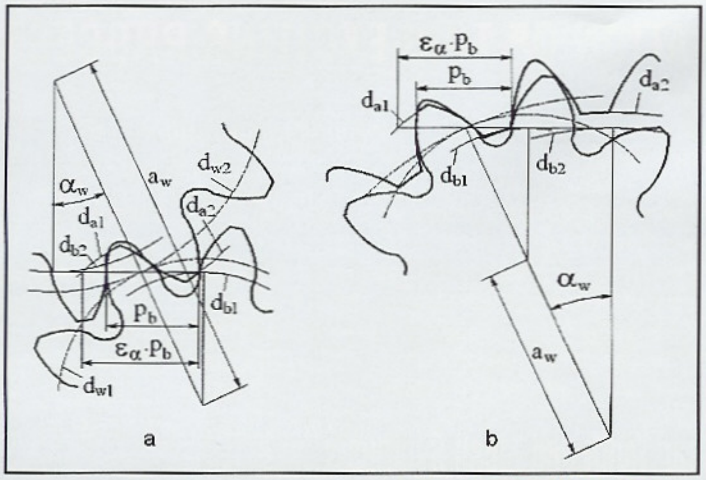

FIG 2: GEAR MESH; a—EXTERNAL GEARING; b—INTERNAL GEARING; a_w—CENTER DISTANCE; p_b—BASE CIRCLE PITCH; α_w—OPERATING PRESSURE ANGLE; ε_α—CONTACT RATIO; $d_{w1,2}$—OPERATING PITCH CIRCLE DIAMETERS; SUBSCRIPTS "1" AND "2" ARE FOR THE MATING PINION AND THE GEAR.

In Direct Gear Design the fillet profile is optimized in order to minimize bending stress concentration. The initial fillet profile is a trajectory of the mating gear tooth tip in the tight (zero backlash) mesh. The FEA and random search method are used for fillet optimization [5]. The calculation process results with forming the optimized fillet profile that provides minimum achievable bending stress. This fillet provides the minimized radial clearance with the mating gear tooth, excluding interference at worst tolerance combination and operating conditions. It also has the maximized curvature radius, distributing the bending stress along a large portion of the fillet, reducing stress concentration (Fig. 3). The shape of the optimized fillet profile depends on the mating gear geometry. It would be very different if the same gear were in mesh the external gear, or the internal gear, or the gear rack. However, it practically does not depend on the load level and load application point. If the gear is in mesh with several different gears, like in the planetary stage for example, its profile is optimized to exclude interference with any gear.

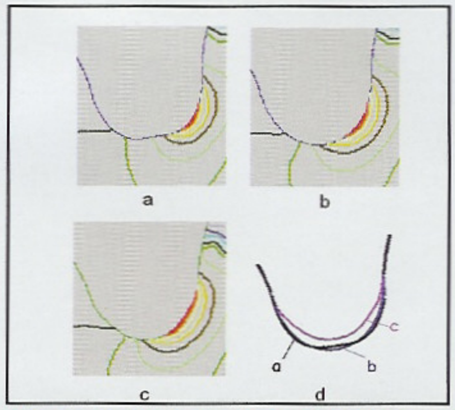

FIG 3: BENDING STRESS DISTRIBUTION ALONG THE FILLET; A—THE STANDARD AGMA 201.02 20 DEGREE RACK GENERATED FILLET; B—THE FULL RADIUS 20 DEGREE RACK GENERATED FILLET; C—THE OPTIMIZED FILLET; D - THREE FILLET OVERLAY.

Table 1 presents bending stress reduction, achievable by the full radius rack application and by the fillet profile optimization in comparison to the standard 20°

TABLE 1

Pinion and gear number of teeth	Bending stress reduction in comparison with the standard 20° rack, %		Bending stress reduction in comparison with the standard 25° rack, %	
	Full radius 20° rack	Optimized fillet	Full radius 25° rack	Optimized fillet
12	-	-	8	21
15	6	25	7	20
20	10	23	6	18
30	10	21	6	17
50	10	21	5	15
80	10	21	5	14
120	10	21	4	13

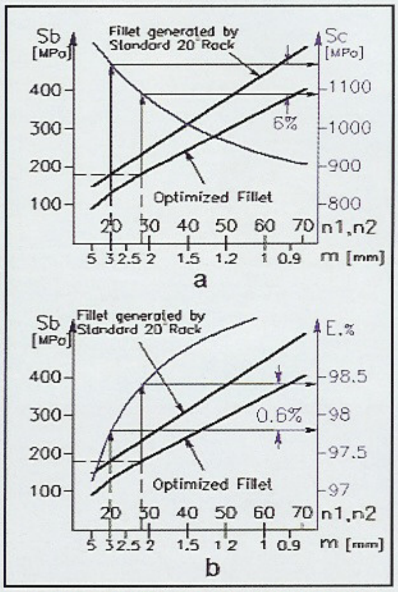

FIG. 4: A—CONTACT STRESS REDUCTION; B—INCREASED MESH EFFICIENCY.

and 25° rack for gears with different number of teeth. The involute portion of the tooth profile is the same.

The graphs in Fig. 4 present stresses and efficiency of the gears with different fillets, the constant center distance $a_w = 60$ mm, the face width of both gears $b = 10$ mm, and the driving torque $T = 50$ Nm. The graphs show that, if the gears with the standard and optimized fillet have the same acceptable bending stress level, the gears with optimized fillet have finer pitch (smaller module) and higher number of teeth. This results with contact stress reduction, because of the increased contact ratio and increased mesh efficiency.

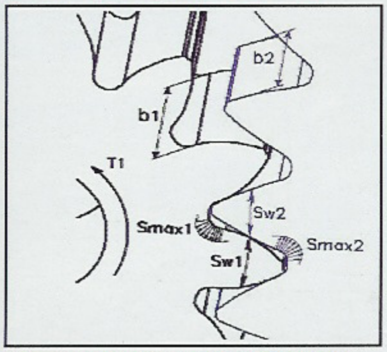

FIG. 5: BALANCE OF THE MAXIMUM BENDING STRESSES

Bending Stress Balance

Mating gears should be equally strong. If the initially calculated bending stresses for the pinion and the gear are significantly different, the bending stresses should be balanced [5].

DGD defines the optimum tooth thickness ratio S_{w1}/S_{w2} (Fig. 3), using FEA and an iterative method, providing a bending stress difference of less than 1 percent. If the gears are made out of different materials, the bending safety factors should be balanced.

Direct Gear Design is applicable for all kind of involute gears: the spur gears including, external, rack and pinion, and external, helical, bevel, worm, face gears, etc. The helical, bevel, and worm gear tooth profile is typically optimized in the normal section. The face gear fillet is different in every section along the tooth line. Therefore its profile is optimized in several sections and then is blended into the fillet surface.

Gears with Asymmetric Teeth

Two profiles (flanks) of a gear tooth are functionally different for many gear drives. The workload on one profile is significantly higher and is applied for longer periods of time than for the opposite one. The design of an asymmetric tooth shape reflects this functional difference. The gears with asymmetric teeth (Fig. 6) are naturally suitable for Direct Gear Design because there are not standards for the asymmetric basic or generating racks. The design intent of asymmetric gear teeth is to improve performance of the primary drive profiles by some degrading performance of the opposite coast profiles. The coast profiles are unloaded or lightly loaded during relatively short work period.

The main advantage of asymmetric gears is contact stress reduction, resulting higher torque density (load capacity per gear size). Another important advantage is possibility to design the coast flanks and fillet independently from the drive flanks, managing tooth stiffness, and load sharing, while keeping a desirable pressure angle and contact ratio on the drive profiles. This

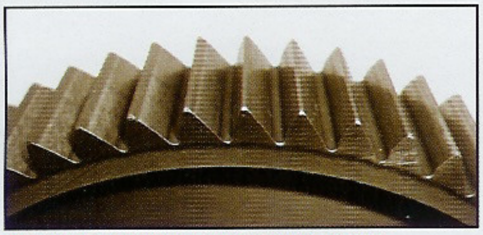

FIG. 6: GEAR WITH ASYMMETRIC TEETH

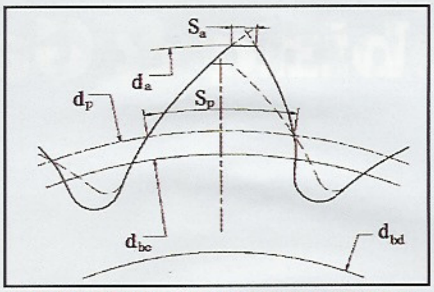

FIG. 7: ASYMMETRIC TOOTH PARAMETERS

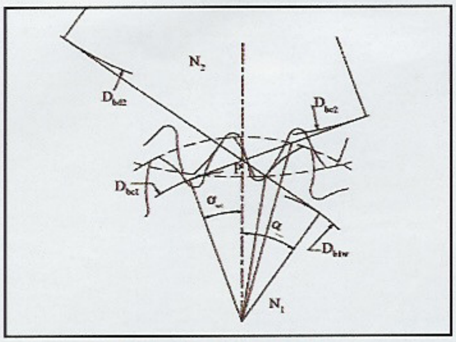

FIG. 8: ASYMMETRIC GEAR MESH

allows reducing gear noise and vibration level.

The DGD approach for asymmetric gears is the same as for symmetric gears. The only difference is that the asymmetric tooth (Fig.7) is defined by two involutes of two different base circles d_{bd} and d_{bc}. The common base tooth

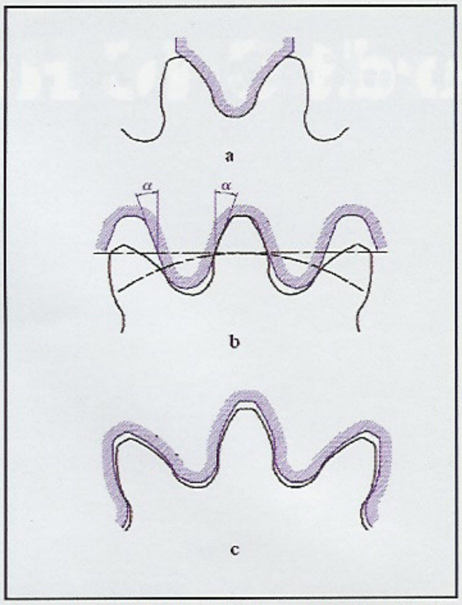

FIG. 9: TOOLING PROFILES; A—PROFILE GEAR MACHINING, B—GENERATING GEAR MACHINING, C—GEAR MOLDING, POWDER METAL PROCESSING, OR CASTING.

thickness does not exist in the asymmetric tooth. The circular distance (tooth thickness) S_p between involute profiles is defined at some reference circle diameter d_p that should be bigger than the largest base diameter.

The asymmetric gear mesh (Fig. 8) presents to two different drive and coast flank meshes

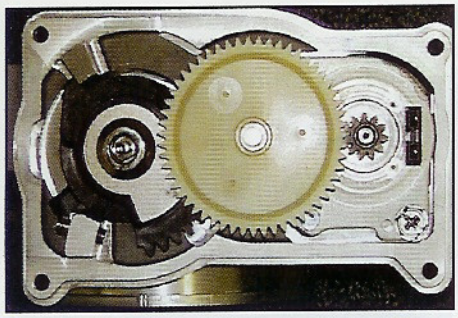

FIG. 10: AN AUTOMOTIVE GEARBOX THAT WORKS IN AN UNDER-THE-HOOD ENVIRONMENT WITH A TEMPERATURE RANGE OF -48°C TO +135°C. INITIALLY, IT WAS DESIGNED WITH MACHINED METAL GEARS, THEN PLASTIC GEARS WERE CONSIDERED FOR COST REDUCTION. THE THERMAL EXPANSION OF THE PLASTIC GEARS, THE POWDER METAL PINION, AND THE ALUMINUM HOUSING ARE VERY DIFFERENT RELATIVE TO EACH OTHER. THE DIRECT DESIGNED GEARS WITH LONG AND FLEXIBLE TEETH ALLOWED FOR THE ABSORPTION OF ALL DIMENSIONAL CHANGES RELATED TO THE VARIABLE OPERATING CONDITIONS.

with different pressure angles and contact ratios. With asymmetric gears it is possible to simultaneously increase the transverse contact ratio and operating pressure angle beyond the conventional gear limits.

Table 2 presents comparison traditionally designed gear pair with the bending stresses balanced by addendum modification, generated by the full radius 25° rack with similar

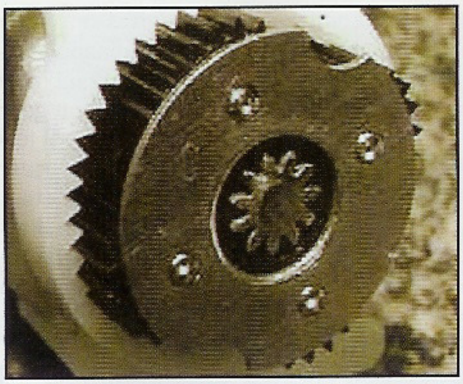

FIG. 11: THE PLANETARY DRIVE FOR AN AUTOMATION APPLICATION. THE GOAL WAS TO PROVIDE A HIGH GEAR RATIO (63:1) IN ONE PLANETARY STAGE AND HIGH OUTPUT TORQUE IN A VERY LIMITED SPACE. THE GEAR ARRANGEMENT OPTIMIZATION IN COMBINATION WITH DGD PROVIDED SUCCESS IN ACHIEVING THIS GOAL.

gear pairs created by Direct Gear Design with different tooth profiles: symmetric, asymmetric, with conventional and high contact ratio. This table illustrates the contact and bending stress reduction as a result of complete tooth profile optimization.

Tooling for Directly Designed Gears

The optimized gear profiles require the custom tooling. For profile machining process (Fig. 9a) the tool profile is the same as a space profile between the neighboring teeth. For generating machining process like gear hobbing (Fig. 9b) the tool profile is defined by reverse genera-

TABLE 2

	Traditional design with full radius 25° rack	Direct Gear Design			
		Symmetric gears	Symmetric gears	Symmetric HCR gears	Symmetric HCR gears
Pinion number of teeth	27	27	27	27	27
Gear number of teeth	49	49	49	49	49
Module, mm	3.0	3.0	3.0	3.0	3.0
Drive pressure angle, °	25	25	25	20	24
Coast pressure angle, °	25	25	25	20	16
Drive contact ratio	1.5	1.5	1.5	2.06	2.06
Pinion torque, Nm	300	300	300	300	300
Pinion Bending Stress, MPa	196	167	171	128	130
Gear Bending Stress, MPa	198	167	171	125	128
Contact Stress, MPa	976	976	887	822	777

tion, when the designed gear forms the tooling rack profile. The pressure angle, in this case, is selected to provide better machining conditions. For gear molding, powder metal processing, and casting (Fig. 9c) the tool cavity profile is the same as the whole gear profile, adjusted for warpage and shrinkage.

Summary

Now gear designers can decide to use the traditional or the Direct Gear Design method. Traditional gear design is driven by manufacturing, when gear interchangeability, low tool inventory, and design simplicity are important. It is for low expectation gear drives. Typical examples are shelf gears, low production volume machined gears, gear drives with interchangeable gear sets, etc.

Direct Gear Design is driven by application, when technical and market performance of product is critical. It is for custom gear drives. Typical examples are the molded, forged, cast, and powder metal gears. The mass production machined gears, gears for critical and extreme applications, like aerospace, automotive, and racing gear transmissions.

FIG. 12: THESE CROSSED HELICAL GEARS ARE USED IN AGRICULTURAL EQUIPMENT. IT WAS POSSIBLE TO REPLACE AN OLD-STYLE METAL CHAIN DRIVE WITH A LESS EXPENSIVE, SAFE, AND MAINTENANCE-FREE PLASTIC GEAR DRIVE AND FLEXIBLE SHAFT.

FIG. 13: THIS MEDICAL PUMP DEMONSTRATES THE DIRECTLY DESIGNED FLEXIBLE ASYMMETRIC TEETH THAT PROVIDE A BETTER SEAL BETWEEN THE GEARS AND THE HOUSING, RESULTING IN HIGHER OUTPUT PRESSURE, FLOW, AND EFFICIENCY.

FIG. 14: IN THESE GENERATOR GEARS, APPLICATION OF THE ASYMMETRIC FOR THE EXPERIMENTAL GENERATOR GEAR DRIVE ALLOWED FOR A SIGNIFICANTLY REDUCED VIBRATION LEVEL.

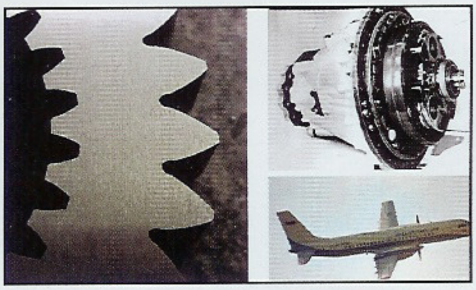

FIG. 15: THE TWO-STAGE PLANETARY GEAR REDUCER OF THE TURBO-PROP ENGINE TV7-117 (DESIGN BUREAU NAMED AFTER V.Y. KLIMOV, ST. PETERSBURG, RUSSIA) FOR THE TWO-ENGINE ILYUSHIN-114 PLANE (CURRENTLY IN PRODUCTION) HAS ALL GEARS WITH ASYMMETRIC TEETH. APPLICATION OF THE ASYMMETRIC TEETH HELPED TO PROVIDE VERY LOW WEIGHT TO OUTPUT TORQUE RATIO AND CUT DOWN DURATION AND EXPENSE OF OPERATIONAL DEVELOPMENT.

Conclusion

The presented alternative Direct Gear Design method provides complete gear tooth profile optimization resulting significant contact and bending stress reduction. This stress reduction is converted to:

- Higher load capacity
- Reduced size and weight
- Extended lifetime
- Reduced noise and vibration
- Higher efficiency
- Higher reliability
- Reduced cost

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