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# DIRECT DESIGN OF ASYMMETRIC GEARS: APPROACH AND APPLICATION

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## INTRODUCTION

In propulsion gear transmissions the tooth load on one flank is significantly higher and is applied for longer periods of time than for the opposite one. An asymmetric tooth shape reflects this functional difference. Design intent of asymmetric gear teeth is to improve performance of the primary drive profiles at the expense of the performance for the opposite coast profiles. The coast profiles are unloaded or lightly loaded during a relatively short work period. Asymmetric tooth profiles also make it possible to simultaneously increase the contact ratio and operating pressure angle beyond the conventional gears' limits. The main advantage of asymmetric gears is contact stress reduction on the drive flanks, resulting in higher torque density (load capacity per gear size). These benefits of the gears with asymmetric tooth profiles for unidirectional torque transmission are well known. There are many publications on this subject written by authors from different countries [1 - 8]. All these and other related articles, however, are dedicated to the mathematical modeling or lab specimen testing and don't present any practical implementations of the gears with asymmetric tooth profiles.

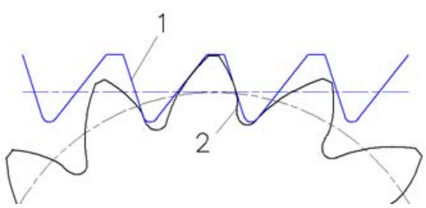


Fig 1. Asymmetric rack generating gear design; 1 – pre-selected rack (cutter) profile, 2 – gear profile

In most of these researches [2 – 4, 7,8] the asymmetric teeth profiles are defined by the pre-selected asymmetric generating gear rack parameters (see Fig. 1), - a common method for conventional gears with symmetric teeth. Advantages of this method for symmetric gears include interchangeability, wide availability of standard tooling, and great analytical and testing database for the gears with symmetric teeth. However, all these advantages are not applicable to the gears with asymmetric teeth, because the standards for this kind of gears do not exist and analytical and testing database is not sufficient for practical definition of the asymmetric rack that could be considered for standardization.

This paper presents the non-traditional Direct Gear Design® (DGD) method of the asymmetric gears. It also demonstrates an example of the gearbox where the asymmetric gears are used which passed all development stages and currently is in production.

## GEAR GEOMETRY

### Gear tooth parameters

DGD is a practical realization of the Gearing Theory of Generalized Parameters developed by Prof. E.B. Vulgakov [1]. He also has applied this theory to the gears with asymmetric involute teeth, presenting the asymmetric tooth as a combination of two halves of different symmetric teeth without using any generating rack. Later A.L. Kapelevich [5,6] defined the asymmetric tooth by two involutes from different base diameters that drastically expanded previously achievable range of the gear mesh parameters, such as the operating pressure angle and contact ratio.

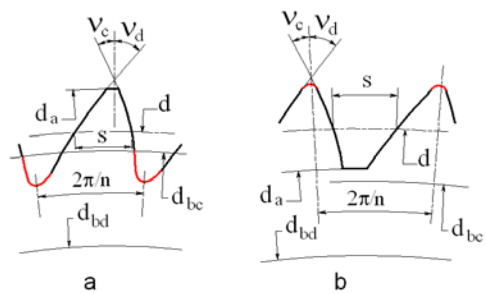


Fig 2. Asymmetric tooth profile (the fillet portion is red); a – external tooth; b – internal tooth; $d_a$ – tooth tip circle diameter; $d_b$ – base circle diameter; d – reference circle diameter; S – circular tooth thickness at the reference diameter; ν - involute intersection profile angle; subscripts "d" and "c" are for the drive and coast flanks of the asymmetric tooth

DGD presents the asymmetric tooth form by two involutes of two different base circles with the arc distance between them and tooth tip circle describe the gear tooth (Fig.2). The equally spaced teeth form the gear. The fillet between teeth is not in contact with the mating gear teeth. However, this portion of the tooth profile is critical because this is the area of the maximum bending stress concentration. The fillet profile is designed independently and a subject of optimization providing minimum bending stress concentration and sufficient clearance with the mating gear tooth tip in mesh.

### Gear mesh parameters

The asymmetric gear mesh (Fig. 3) presents to two different drive and coast flank meshes with different pressure angles and contact ratios.

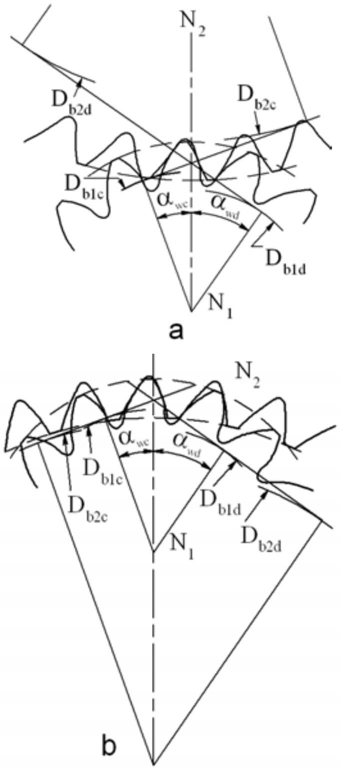


Fig 3. Asymmetric gear mesh; a – external gearing; b – internal gearing; $\alpha_w$ - operating pressure angle; $d_{b1,2}$ – operating pitch circle diameters; subscripts "1" and "2" are for the mating pinion and the gear.

The operating pressure angle $\alpha_w$ and the contact ratio $\varepsilon_\alpha$ for the gear with asymmetric teeth are defined by the following formulae:

- for external gearing

$$\text{inv}\ \alpha_{wd} + \text{inv}\ \alpha_{wc} = [\text{inv}\ \nu_{1d} + \text{inv}\ \nu_{1c} + u * (\text{inv}\ \nu_{2d} + \text{inv}\ \nu_{2c}) - 2 * \pi / n_1)] / (1 + u),$$

$$\varepsilon_{\alpha d} = n_1 * [\tan \alpha_{a1d} + u * \tan \alpha_{a2d} - (1 + u) * \tan \alpha_{wd}] / (2 * \pi),$$

$$\varepsilon_{\alpha c} = n_1 * [\tan \alpha_{a1c} + u * \tan \alpha_{a2c} - (1 + u) * \tan \alpha_{wc}] / (2 * \pi).$$

- for internal gearing

$$\text{inv}\ \alpha_{wd} + \text{inv}\ \alpha_{wc} = [u * (\text{inv}\ \nu_{2d} + \text{inv}\ \nu_{2c}) - \text{inv}\ \nu_{1d} - \text{inv}\ \nu_{1c}] / (u - 1)],$$

$$\varepsilon_{\alpha d} = n_1 * [\tan \alpha_{a1d} - u * \tan \alpha_{a2d} + (u - 1) * \tan \alpha_{wd}] / (2 * \pi),$$

$$\varepsilon_{\alpha c} = n_1 * [\tan \alpha_{a1c} - u * \tan \alpha_{a2c} + (u - 1) * \tan \alpha_{wc}] / (2 * \pi),$$

where $u = n_2 / n_1$ is a gear ratio, $\alpha_a = \text{acos}(d_b/d_a)$ - involute profile angle at the tooth tip circle diameter.

**Tooth fillet profile optimization**

The tooth fillet design begins when the involute flank parameters are completely defined. The initial fillet profile is a trajectory of the mating gear tooth tip in the tight (zero backlash) mesh.

The fillet optimization process [9] utilizes three methods:

- random search method locating fillet points;
- trigonometric functions for fillet profile approximation;
- FEA for stress calculation.

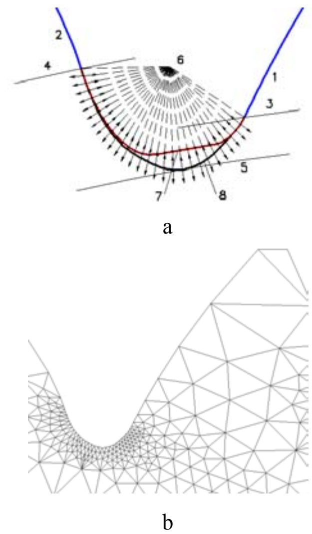


Fig 4. Fillet profile optimization; a – random search node location; 1 and 2 – drive and coast involute tooth flanks, 3 and 4 – drive and coast form circle diameter, 5 – root diameter, 6 – fillet center, 7 – initial fillet profile, 8 - optimized fillet profile; b – FEA mesh

The first and the last fillet profile points of the initial fillet profile lay on the form diameter circle (Fig. 4) and cannot be moved during an optimization process. The random search method is used to move the fillet nodes along the beams that pass through the approximately defined fillet center and the nodes of the initial fillet profile. The bending stresses are calculated for every new fillet profile points' combination. If the maximum bending stress is reduced the program continues searching in the same direction, if not it steps back and starts searching the different direction. After the given number of iteration steps (or given optimization time) the optimization process stops, resulting with an optimized fillet profile that provides minimum bending stress concentration.

The Fig. 5 and Table 1 presents a comparison of the rack generating fillets with the optimized fillet, which provides minimum bending stress concentration.

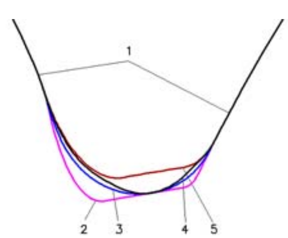


Fig 5. Different fillets of the asymmetric tooth; 1 – involute flanks; 2 - fillet profile (pink) generated by the flat tip rack, 3 – fillet profile

(blue) generated by the full tip radius rack; 4 – optimized fillet profile (black); 5 – trajectory of the mating gear tooth tip in tight mesh (red)

Table 1

| | | |
|---|---|---|
| Module, mm | | 5.6 |
| Number of Teeth | | 25 & 25 |
| Pressure Angle,° | Drive Flank | 40 |
| | Coast plank | 18 |
| Face Width, mm | | 70 & 70 |
| Driving Torque, Nm | | 2700 |
| Maximum Bending Stress, MPa | Flat Tip Generating Rack Fillet | 323(+30%) |
| | Full Radius Tip Rack Fillet | 249 (0%) |
| | Optimized Fillet | 225(–10%) |

## ASYMMETRIC VS. SYMMETRIC GEARS

Table 2 presents a comparison of traditionally designed gear pairs with conventional and high gear ratio, generated by the full radius 25° and 20° racks accordingly with similar gear pairs created by Direct Gear Design with asymmetric teeth.

Table 2

| Gears | Conventional contact ratio | | High contact ratio | |
|---|---|---|---|---|
| Tooth shape | Symmetric (Rack generated) | Asymmetric (DGD) | Symmetric (Rack generated) | Asymmetric (DGD) |
| Number of teeth | 27 (pinion)<br>49 (gear) | | | |
| Module, mm | 3.0 | | | |
| Face Width, mm | 32 (pinion)<br>30 (gear) | | | |
| Pressure angle, ° | 25*/ 25** | 32*/ 18** | 20*/ 20** | 24*/ 16** |
| Contact ratio | 1.47* | 1.47* | 2.06* | 2.06* |
| Pinion torque, Nm | 300 | | | |
| Maximum Bending Stress, MPa | 196 (pinion) 198 (gear) | 171 (pinion) 171 (gear) | 151 (pinion) 153 (gear) | 130 (pinion) 128 (gear) |
| Contact Stress, MPa | 976 | 887 | 822 | 777 |

* for drive tooth flanks;
** for coast tooth flanks.

## EXAMPLE OF IMPLEMENTATION

An example of implementation of the gears with asymmetric tooth profiles is the two stage planetary gearbox of the TV7-117S turboprop engine [9, 10]. This engine has been used in the Russian airplane IL-114 for several years and is going to be used in IL-112, MIG-110, TU-136 airplanes.

### Gearbox data and arrangement

In conventional planetary gear systems, the planet gear is usually in simultaneous contact with the sun and planet gears. The tooth load and number of load cycles are equal for both flanks of the planet gear. However, one flank of the planet gear is in mesh with the concave tooth flank of the ring gear with internal teeth. Resulting contact stress in this mesh is much lower in comparison with contact stress of the convex tooth flanks in sun-planet gear contact, which defines the load capacity and size of the gears. In order to reduce this contact stress the higher operating pressure angle was chosen for the sun-planet gear contacting tooth flanks. This choice is in compliance with the ANSI/AGMA 6123-B06 standard “Design Manual for Enclosed Epicyclic Gear Drives”, which states: “Best strength to weight ratio is achieved with high operating pressure angles at the sun to planet mesh and low operating pressure angles at the planet to ring gear mesh”.

The drive tooth flanks of the sun-planet gear mesh have increased the contact curvature radii resulting with the greater hydrodynamic oil film thickness, lower flash temperature and scuffing probability.

Main parameters of the TV7-117S gearbox are presented in the Table 3

Table 3

| | |
|---|---|
| Input Turbine RPM | 17500 |
| Output Prop RPM | 1200 |
| Total Gear Ratio | 14.6:1 |
| Overall Dimensions, mm:<br>- Diameter<br>- Length | <br>520<br>645 |
| Gearbox weight, N | 1050 |
| Cruse Transmitted Power, hp | 2500 |
| Maximum Transmitted Power, hp | 4000 |

The TV7-117S gearbox arrangement (Fig.7) is proved to provide maximum power transmission density for the required total gear ratio.

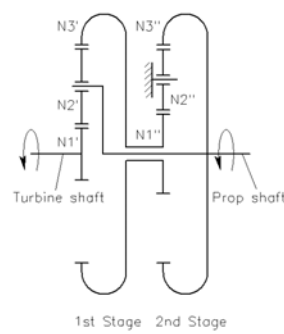


Fig 6. TV7-117S Gearbox arrangement

The first planetary-differential stage has three planet gears. The second coaxial stage has five planet (idler) gears and stationary carrier. Part of transmitted power goes from the first stage carrier directly to the propeller shaft. The rest of the transmitted power goes from the first stage ring gear to the second stage sun gear and then through the planets to the second stage ring gear also connected to the propeller shaft.

### Gear parameters

Basic gear geometry parameters are presented in the Table 4.

Table 4

| Gear | | Sun Gear | Planet Gear | Ring Gear |
|---|---|---|---|---|
| Number of Gears | | 1 | $3^{(1)}, 5^{(2)}$ | 1 |
| Number of Teeth | | $28^{(1)}$ $38^{(2)}$ | $41^{(1)}$ $31^{(2)}$ | $107^{(1)}$ $97^{(2)}$ |
| Operating Pressure Angle, deg. | Drive Flank | 33 | | 29.9 |
| | Coast Flank | 25 | | 36.66 |
| Drive Flank Operating Contact Ratio | | 1.29 | | 1.46 |

$^{(1)}$ - first stage, $^{(2)}$ - second stage

Direct Gear Design of the asymmetric tooth profiles also allows shaping the coast flanks and fillet independently from the drive flanks, managing the tooth bending strength and stiffness, and improving load sharing while keeping a desirable pressure angle and contact ratio on the drive profiles. This allows increasing tooth tip deflection, damping tooth mesh impact, leading to gear noise and vibration reduction.

Number of teeth is selected in order to increase the drive operating pressure angle in the planet-ring gear mesh in comparison with the coast operating pressure angle in the sun-planet gear mesh:

$$\alpha_{wd\ 2\text{-}3} = \arccos [\cos (\alpha_{wc\ 1\text{-}2}) * (n_3 - n_2) / (n_1 + n_2)],$$

Where

$\alpha_{wc\ 1\text{-}2}$ - coast operating pressure angle in the sun-planet gear mesh;

$n_1$ – sun gear number of teeth;

$n_2$ – planet gear number of teeth;

$n_3$ – ring gear number of teeth.

**Gear manufacturing and assembly**

All gears are made of the forged blanks of the steel 20KH3MVF (EI-415). Its chemical composition includes: Fe - base material, C - 0.15-0.20%, S - <0.025%, P - <0.030%, Si - 0.17-0.37%, Mn - 0.25 - 0.50%, Cr - 2.8-3.3%, Mo - 0.35-0.55%, W - 0.30-0.50%, Co - 0.60-0.85%, Ni - < 0.5%.

Machining of the directly designed sun and planet gears with asymmetric teeth requires custom gear cutters. The gear hob rack profile and the shaper cutter profile and are defined by reversed generation of the gear profile. The gear blanks position during machining must provide the asymmetric teeth point in a certain direction (clockwise or counterclockwise). Otherwise, the drive flank of one gear will be positioned in contact with the coast profile of the mating gear that makes the assembly impossible.

After preliminary tooth cutting, the gears are carburized and heat-treated to achieve tooth surface hardness > 59 HRC with the case depth of 0.6-1.0 mm. The core tooth hardness is 33-45 HRC.

Final gear machining includes tooth grinding and honing. Asymmetric gear flanks also require special setup for both these operations.

Assembly of the gearbox includes selection of planet gears and their initial orientation, which is based on transmission error function of every gear. All planet gears are classified by transmission error (TE) function in several groups. Each group has planet gears with similar TE function. Then during assembly, position and orientation of the each planet gear depends on its TE function profile, providing better engagement of the driving flanks and load distribution between planet gears [11].

Some of the TV7-117S turboprop engine gearbox components and assemblies are presented in the Fig. 7 – 10.

Fig 7. First stage sun gear assembly

Fig 8. First stage carrier and ring gear assembly

Fig 9. Second stage sun gear

Fig 10. Second stage carrier assembly


## SUMMARY

This paper describes an alternative Direct Gear Design approach for the asymmetric gear design, demonstrating the basic gear tooth and mesh parameter definition. It also familiarizes with a proprietary tooth fillet profiles optimization technique, providing minimum bending stress concentration.

Application of the asymmetric gears in the TV7-117S turboprop engine gearbox resulted in extremely low weight to output torque ratio, about 50% lower in comparison with the gearboxes of its predecessors AI-20 and AI-24 turboprop engines. It also significantly reduced noise and vibration level, cut down duration and expense of operational development.

The new design and technological approaches that have found their realization in the TV7-117S engine gearbox were recommended for the development of gear boxes for advanced aviation engines.


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## NOMENCLATURE

$d$ – reference circle diameter;
$d_a$ – tooth tip circle diameter;
$d_b$ – base circle diameter;
$n$ – number of teeth;
$S$ – tooth thickness at reference circle diameter;
$u$ – gear ratio;
$\alpha_a$ – involute profile angle at the tooth tip circle diameter;
$\alpha_w$ – operating pressure angle;
$\varepsilon_\alpha$ – contact ratio;
$\nu$ – involute intersection profile angle;
subscripts "1" and "2" are for the pinion and gear;
subscripts "d" and "c" are for the drive and coast flanks of the asymmetric tooth.