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Direct Gear Design® – for Optimal Gear Performance

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The paper presents the Direct Gear Design – an alternative method of analysis and design of involute gears, which separates gear geometry definition from tool selection, to achieve the best possible performance for a particular product and application.

1. Direct Gear Design Overview.

The direct design approach, which uses the operating conditions and performance parameters as a foundation for the design process, is common for most parts of mechanisms and machines (for example, cams, compressor or turbine blades, pump rotors, etc. (See Fig.1).

Fig. 1

Ancient engineers successfully used Direct Gear Design. They were aware of the desirable performance parameters such as a gear ratio, center distance and available power source (water current, wind, horse power). They used them to define the gear parameters (See Fig.2): diameters, number and shape of the teeth for each gear. Then they manufactured gears and carved their teeth using available materials, technology, and tools.

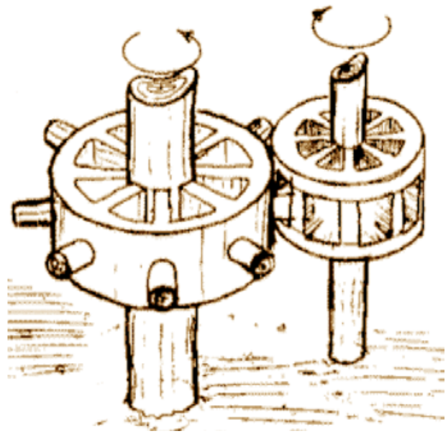


Fig.2

It is important to note that the gear and tooth geometry were defined (or designed) first. Then the manufacturing process and tools were forming or cutting this geometry in wood, stone, or metal. In other words, gear parameters were primary and manufacturing process and tool parameters were secondary. This is the essence of Direct Gear Design.

During the technological revolution in the 19th century, the gear generating process was developed. This process uses a gear rack profile as a cutting edge of the hob that is in mesh with the gear blank (Fig.3).

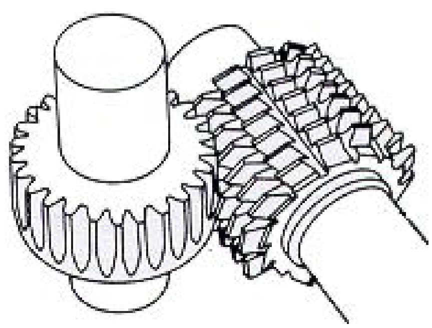


Fig.3

Gear hobbing was a reasonably accurate and highly productive manufacturing process. With some exceptions, gears that are cut by the same tool can mesh together. Hobbing machines required complicated and expensive tools. Common parameters of the cutting tool (generating rack) such as profile (pressure) angle, diametral pitch, tooth addendum and dedendum (Fig.4) were standardized and became the foundation for gear design. This has made gear design **indirect,** depending on pre-selected (typically standard) set of cutting tool parameters.

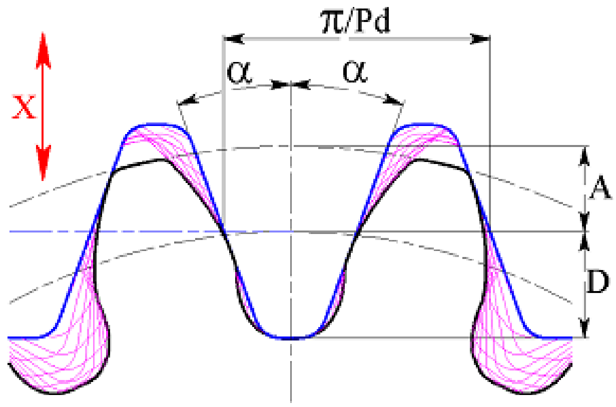


Fig.4

This “traditional” gear design approach has its benefits:

- interchangeability of the gears;
- low tool inventory;
- simple (like fastener selection) gear design process.

When the tool is chosen, there is only one way to affect the gear tooth profile: positioning the tool relative to the gear blank. This will change the tooth thickness, root diameter, outer diameter, and strength of the tooth as a result. This tool positioning is called addendum modification or X – shift. It is used to balance the gear strength, reduce sliding, etc.

Traditional gear design based on standard tool parameters provides “universality” - acceptable for many gear applications. At the same time, it doesn’t provide the best possible performance for any particular gear application because it is self-constrained with predefined tooling parameters.

Traditional tool based gear design is not the only available approach to design gears. There is another approach - the Direct Gear Design.

Modern Direct Gear Design is based on the gear theory of generalized parameters created by Prof. E.B. Vulgakov.

> **<u>Direct Gear Design</u> is an application driven gear development process with primary emphasis on performance maximization and cost efficiency without concern for any predefined tooling parameters**

2. Gear Mesh Synthesis.

Direct Gear Design defines the gear tooth without using the generating rack parameters like diametral pitch, module, or pressure angle. The gear tooth (Fig.5) is defined by two involutes of base circle d_b and the circular distance (base tooth thickness) S_b between them. The outer diameter d_a limits tooth height to avoid having a pointed tooth tip and provides a desirable tip tooth thickness S_a. The non-involute portion of the tooth profile, the fillet, does not transmit torque, but it is a critical element of the tooth profile. The fillet is an area with the maximum bending stress, which limits the strength and durability of the gear.

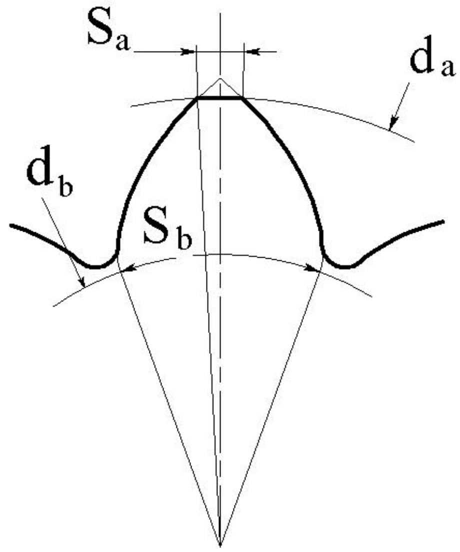


Fig.5

Two involute gears can mesh together (Fig.6), if they have the same base circle pitch. Other parameters of a gear mesh are:

- center distance aw;
- operating pitch diameters d_{w1} and d_{w2} (diameters with pure rolling action and zero sliding);
- tooth thicknesses on the operating pitch diameters S_{w1} and D_{w2};
- operating pressure angle α_w (involute profile angle on the operating pitch diameters);
- contact ratio ε_α.

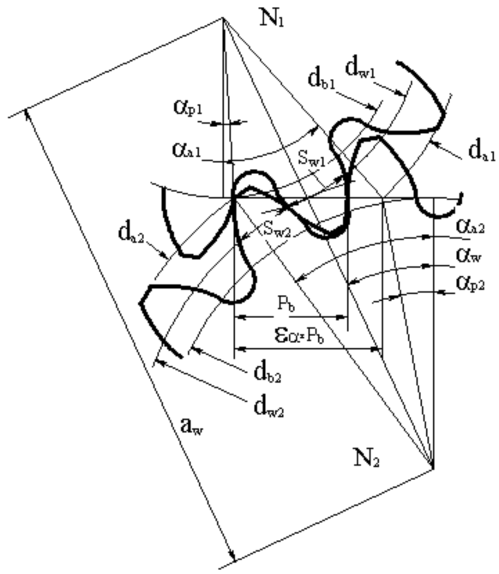


Fig.6

There is a principal difference in the pressure angle definitions in traditional and Direct Gear Design. In traditional gear design the pressure angle is the tooling rack profile angle. In Direct Gear Design the pressure angle is the mesh parameter. It does not belong to one gear. If the mesh condition (the center distance, for example) is changed, the pressure angle is changed as well.

Direct Gear design is applicable for all kinds of involute gears: spur, helical, bevel, worm, and others (Fig. 7).

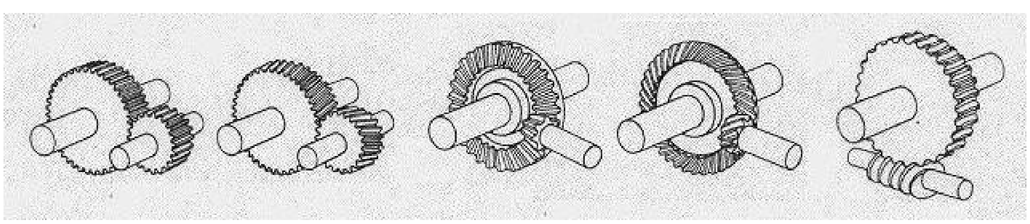


Fig.7

The normal section of these gears can be replaced with the virtual spur gears. The virtual spur gears have the same normal section profile as the real gears, but different number of teeth. There is an assumption that relative improvement of the spur virtual gears, leads to improvement of the real gear.

The following formulas are used to define the virtual numbers of teeth:

- for helical gears $N_v = N/\cos(\beta)^3$, N is the real number of teeth, β is the operating helix angle;
- for straight bevel gears $N_v = N/\cos(\gamma)$, γ is the operating cone angle;
- for spiral bevel gears N_v v $= N/\cos(\gamma)/\cos(\beta)^3$;
- for worm gears $N_{wv} = N_w/\cos(90^o\text{-}\beta)^3$ and $N_{wgv} = N_{wg}/\cos(\beta)^3$, N_w is the number of starts of the worm, N_{wg} is the number of teeth of the worm gear.

Direct Gear Design input data:

Nominal Operating Diametral Pitch

Nominal Operating Pressure Angle (for gears with asymmetric teeth the Nominal Operating Pressure Angles are different for drive side and coast side of the teeth).

Pinion Torque.

Friction Coefficient.

Drive side Contact Ratio.

Numbers of teeth, tip radii, and face widths for the pinion and the gear.

The pinion and the gear material properties: Modulus of Elasticity and Poisson Ratio.

Initial Pitch Diameter Tooth Thickness Ratio (the pinion tooth thickness divided on the gear tooth thickness).

Output data:

All gear geometry parameters (diameters, profile angles, and tooth thicknesses), specific sliding velocities, gear efficiency, and geometrical and load data for the Finite Element Analysis (FEA).

3. Efficiency Maximization

Gear efficiency maximization is important not only for high speed and high loaded gear drives. In gear transmissions almost all inefficiency or mechanical losses is transferred to heat reducing gear performance, reliability, and life. This is especially critical for plastic gears. Plastics do not conduct heat as well as metal. Heat accumulates on the gear tooth surface leading to premature failure.

The gear efficiency for spur (or virtual spur) gears is

$$E := 100\cdot\left[1 - \frac{f}{2\cdot\cos(\alpha)}\cdot\frac{(H_1)^2 + (H_2)^2}{H_1 + H_2}\right]\%$$

Where

H_1 and H_2 are maximum specific sliding velocities of the pinion and the gear;

f is friction coefficient;

α is operating pressure angle.

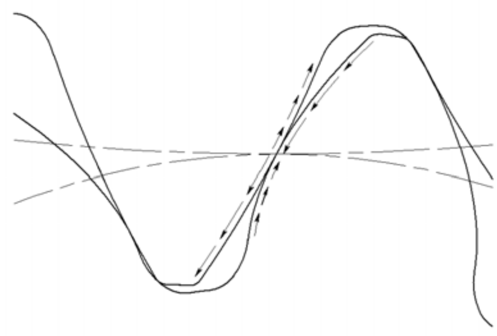


Fig.8

Direct Gear Design maximizes gear efficiency by equalizing maximum specific sliding velocities for both gears (Fig. 8). Unlike in traditional gear design, it can be done without compromising gear strength or stress balance.

4. Bending Stress Balance

Next steps of the gear mesh synthesis are the FEA modeling and maximum bending stress evaluation. The FEA is used for the stress calculation because the Lewis equation doesn't provide reliable results for direct designed gears. If initially calculated bending stresses for the pinion and the gear are significantly different, the bending stress balance should be done.

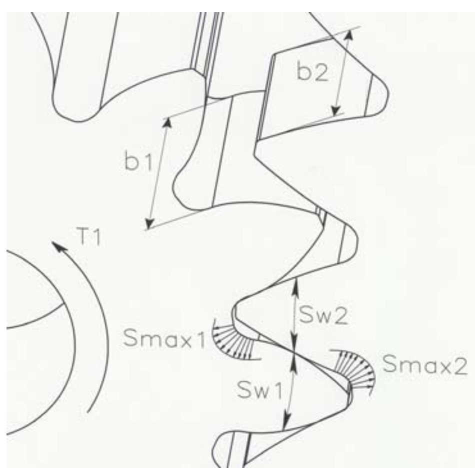


Fig.9 Balance of the bending stresses

The Direct Gear Design defines the optimum tooth thickness ratio S_{p1}/S_{p2} (Fig.9), using the 2D FEA and an iterative method, providing a bending stress difference of less than 1%. If the gears are made out of different materials, the bending safety factors should be balanced.

5. Fillet profile optimization.

Traditional gear design is based on predefined cutting tool parameters and the fillet is determined as a trace of the tool cutting edge. The cutting tool typically provides the fillet profile with an increased radial clearance in order to avoid root interference for a wide range of gears with different numbers of teeth and different addendum modifications that could be cut with this tool. It results in relatively high teeth with small fillet radii in the area of maximum bending stress.

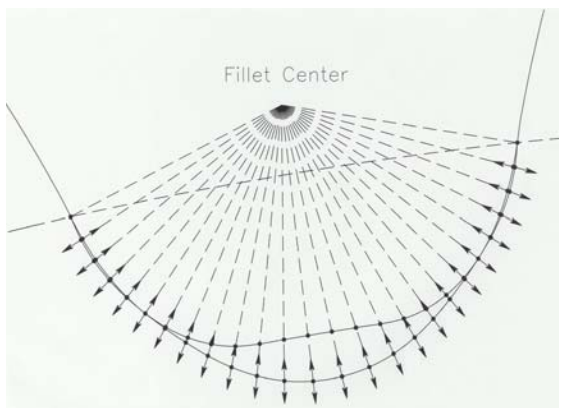
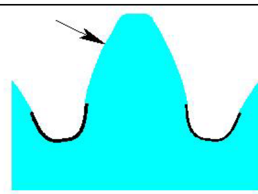
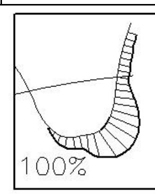
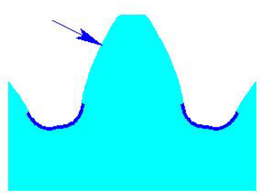
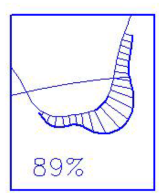
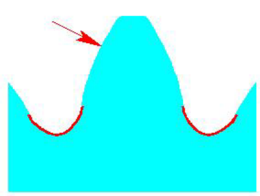
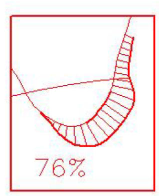


Fig.10

Direct Gear Design optimizes the fillet profile for any pair of gears in order to minimize the bending stress concentration. Initially the fillet profile is a trace of the mating gear tooth tip. The optimization process is based on the 2D FEA and the random search method (Fig.10). The computer program sets up the center of the fillet and connects it with the FEA nodes on the fillet. Then it moves all the nodes along the beams and calculates the bending stress. The nodes cannot be moved above the initial fillet profile because it will lead to interference with the mating gear tooth. The program analyzes successful and unsuccessful steps, finding the direction of altering the fillet profile to reduce the maximum bending stress. This process continues for a certain number of iterations resulting with the optimized fillet profile.

The Table 1 illustrates the fillet profile optimization and the achievable maximum bending stress reduction for the standard (AGMA 201.2) gears.

Table 1

<table>
<tr><td colspan="4">Tool Rack Parameters</td></tr>
<tr><td>Diametral Pitch</td><td colspan="3">10</td></tr>
<tr><td>Pressure Angle</td><td colspan="3">25°</td></tr>
<tr><td>Addendum</td><td colspan="3">.100</td></tr>
<tr><td>Whole Depth</td><td colspan="3">.225</td></tr>
<tr><td colspan="4">Gear Parameters</td></tr>
<tr><td></td><td colspan="2">Pinion</td><td>Gear</td></tr>
<tr><td>Number of teeth</td><td colspan="2">10</td><td>10</td></tr>
<tr><td>Base Diameter</td><td colspan="2">1.8126</td><td>1.8126</td></tr>
<tr><td>Pitch Diameter</td><td colspan="2">2.000</td><td>2.000</td></tr>
<tr><td>Outer Diameter</td><td colspan="2">2.200</td><td>2.200</td></tr>
<tr><td>Form Diameter</td><td colspan="2">1.833</td><td>1.833</td></tr>
<tr><td>Root Diameter</td><td colspan="2">1.750</td><td>1.750</td></tr>
<tr><td>Tooth Thickness on Pitch Diameter</td><td colspan="2">.1571</td><td>.1571</td></tr>
<tr><td>Face Width</td><td colspan="2">.500</td><td>.500</td></tr>
<tr><td>Tip Radius</td><td colspan="2">.015</td><td>.015</td></tr>
<tr><td>Center Distance</td><td colspan="3">2.000</td></tr>
<tr><td colspan="4">Results</td></tr>
<tr><td>Fillet Profile</td><td>Tooth Profile</td><td>Stress Chart</td><td>Bending Stress</td></tr>
<tr><td>Trajectory of the Tool</td><td></td><td></td><td>12,800 psi</td></tr>
<tr><td>Trajectory of the Mating Gear Tooth</td><td></td><td></td><td>11,400 psi</td></tr>
<tr><td>Optimized Profile</td><td></td><td></td><td>9,800 psi</td></tr>
</table>

Example of the gears with the optimized fillet profile is shown in Fig.11

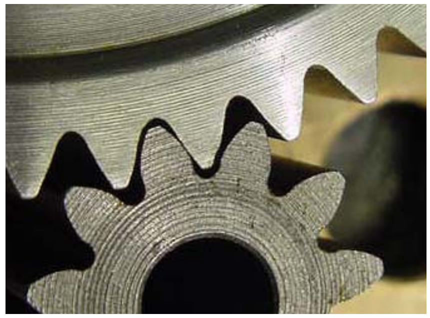


Fig.11

6. Gears with asymmetric teeth

The two profiles (sides) of a gear tooth are functionally different for many gears. The workload on one profile is significantly higher and is applied for longer periods of time than for the opposite one. The design of the asymmetric tooth shape reflects this functional difference (Fig.12).

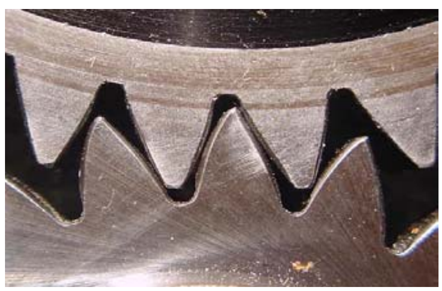


Fig.12

The design intent of asymmetric gear teeth is to improve the performance of the primary contacting profile by degrading the performance of the opposite profile. The opposite profile is typically unloaded or lightly loaded during relatively short work periods.
The degree of asymmetry and drive profile selection for these gears depends on the application.

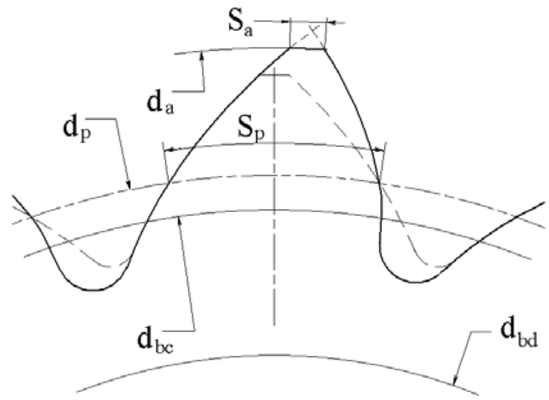


Fig.13

The Direct Gear Design approach for asymmetric gears is the same as for symmetric gears. The only difference is that the asymmetric tooth (Fig.13) is defined by two involutes of two different base circles d_{bd} and d_{bc}. The common base tooth thickness does not exist in the asymmetric tooth. The circular distance (tooth thickness) S_p between involute profiles is defined at some reference circle diameter d_p that should be bigger than the largest base diameter. The mesh of the asymmetric gears is shown in the Fig.14.

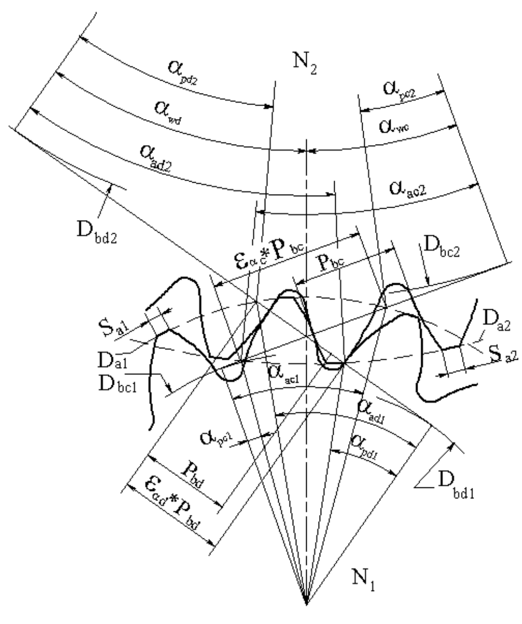


Fig.14

Asymmetric gears simultaneously allow an increase in the transverse contact ratio and operating pressure angle beyond the conventional gear limits. For example, if the theoretical maximum pressure angle for the symmetric spur involute gears is 45^o

pressure, the asymmetric spur gears can operate with pressure angle 50^o - 60^o or higher. Asymmetric gear profiles also make it possible to manage tooth stiffness and load sharing while keeping a desirable pressure angle and contact ratio on the drive profiles by changing the coast side profiles. This provides higher load capacity and lower noise and vibration levels compared with conventional symmetric gears.

7. Tooling and Processing for Direct Designed Gears

The Direct Gear Design approach is dedicated to custom gears and requires custom tooling.

For cut metal gears it means that every gear needs its own hob or shaper cutter. This leads to increased gear cutting tool inventory. The Direct Gear Design approach application must be justified by significantly improved gear performance.

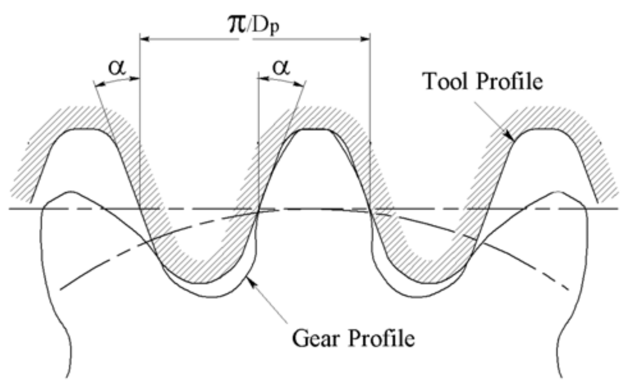


Fig.15

The reversed gear generating process is used to define the generating rack parameters for cutting tool (Fig.15). The gear profile is in mesh with the tool forming its cutting edge. It could be done at different mesh conditions, such as, different pitch diameters and pressure angles. Typically the closest standard pitch is selected. Then the tool pressure angle and other profile parameters are calculated. It allows using standard hobs and just regrinding the cutting edge profile instead of making a whole new tool. The selected tool profile must satisfy the cutting condition requirements such as certain values of the back and side angles of the tool.

The gear machining process for the Direct Designed gears (including gears with asymmetric teeth) is practically the same as that for standard gears.

The plastic gear molding process (as well as gear die casting, gear forging, powder metal gear processing, etc.) doesn't use mesh generation and requires unique tooling for every gear. This makes Direct Gear Design naturally suitable for plastic molded gears because the gear tooth profile customization does not affect the tooling cost, delivery time, or gear processing time.

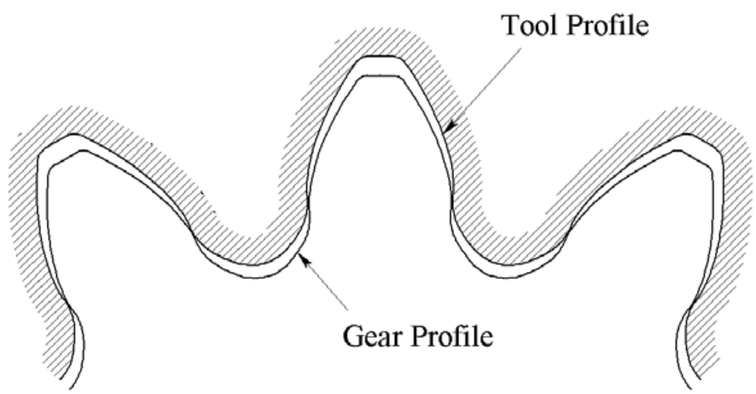


Fig.16

A profile of the plastic gear tool cavity (Fig.16) depends on many factors such as the shape of the gear, material properties, number, size, and location of the gates, molding process parameters, etc. It is practically impossible to predict the tool cavity profile for precision plastic gears in advance. It requires several molding cycles and tool cavity profile adjustments to achieve the required gear accuracy.

AKGears has developed and implemented at Thermotech a proprietary gear cavity adjustment technique called the Genetic Molding Solution®. Dr. Y.V. Shekhtman is created the Genetic Molding Solution software. It is based on the fact that the shape of the molded part contains the genetic information about the material, the tool, and the molding process. The Genetic Molding Solution method stages are illustrated in the Table 2.

Table 2

Genetic Molding Solution method stage	**Comment**
Development of the gear profile data as a result of the Direct Gear Design.	The gear profile data points are presenting the target gear parameters.
Development of the1st tool cavity by simply scaling the gear profile by the material shrinkage factor. Manufacturing the 1st tool cavity.	The 1st tool cavity is needed to define and finalize molding process parameters.
The 1st tool cavity CMM inspection	To confirm the 1st tool cavity is to specification.
Molding of the 1st sample gears and molding process optimization.	Achieving acceptable (consistent, repeatable, fast) molding process and the part material property.
Roll test of the 1st sample gears. Selection of the most representative gear.	There is no concern for the gear quality at this stage. Roll test is required to select the most representative gear with main parameters (TTE, TCE, and the center distance with master gear) in the middle of the process deviation range.
The CMM inspection of the most representative gear.	To collect the 1st sample gear profile data for the final cavity adjustment.
The Genetic Molding Solution® mathematical prediction program application for final cavity profile definition.	The mathematical prediction program uses three data point sets (the designed target gear profile, 1st cavity profile, and the 1st sample gear profile) to calculate the final cavity profile.
Manufacturing and CMM inspection of the final cavity profile.	To confirm the final cavity is to specification.
Molding and roll test inspection of the gears.	To confirm molded gears from the final cavity are to specification.

The Genetic Molding Solution application requires stable material properties, a consistent and repeatable molding process, and reliable inspection. If one of the factors affecting the gear shape is changed (material, process, tool, or molding machine), the Genetic Molding Solution must be applied again. Fig. 17 illustrates the Genetic Molding Solution (GMS) application.

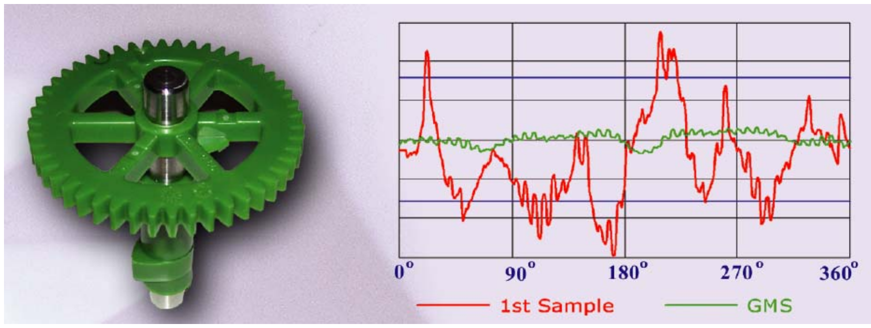


Fig. 17

8. Traditional vs. Direct Gear Design

The Table 3 illustrates differences in basic principles and applications of the Traditional and Direct Gear Design.

Table 3

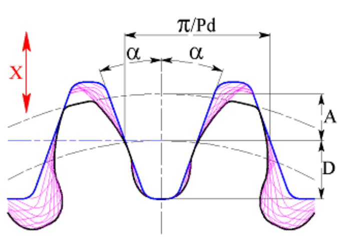
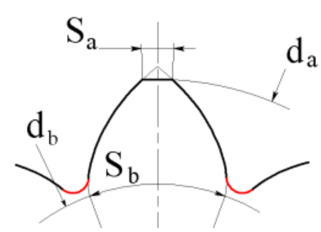
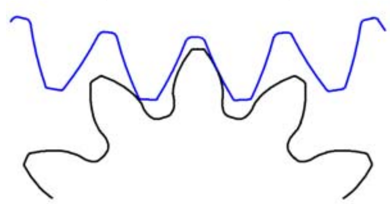
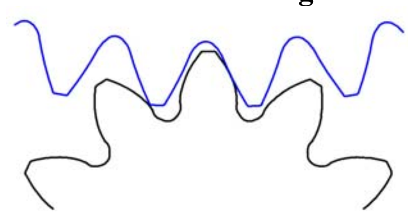
<table>
<tr><th>Traditional Gear Design</th><th>Direct Gear Design</th></tr>
<tr><td></td><td></td></tr>
<tr><th colspan="2">Basic Principle</th></tr>
<tr><td>Gear design is driven by manufacturing (cutting tool profile parameters).</td><td>Gear design is driven by application (performance parameters).</td></tr>
<tr><th colspan="2">Application</th></tr>
<tr><td>General Application Gears
• Stock gears.
• Gearboxes with interchangeable gear sets (like old machine tools).
• Mechanical drive prototyping.
• Low production machined gears.</td><td>Custom Application Gears
• Plastic and metal molded, powder metal, die cast, and forged gears.
• High production machined gears.
• Gears with special requirements and for extreme applications.</td></tr>
</table>

Table 4 presents an example of the direct design gear set in comparison with the “best” traditionally designed gear set based on the 25^{o} pressure angle generating tool. The “best” in this case means the well-balanced gears with minimum bending stresses and relatively high efficiency. Nevertheless, Direct Gear Design results in gears with about 30% lower maximum bending stress, and a higher contact ratio allowing for an increase in the center distance deviation. The gear efficiency is also increased from 97% to 98%, which means 33% less mechanical losses and heat generation resulting in higher reliability and longer life.

Table 4

Shared Attributes:	Pinion		Gear	
Number of teeth	11		57	
Operating Pressure Angle	25°			
Diametral Pitch, 1/in	20			
Center Distance, in	1.338			
Face Width, in	.472		.394	
Pinion Torque, in-lb	14			
Gear Profiles	**The Best Traditional Design (AGMA 201.2)** 		**Direct Gear Design®** 	
Performance Parameters	Pinion	Gear	Pinion	Gear
Max. Bending Stress, psi	8100	8600	5800(-28%)	6000(-30%)
Contact Ratio	1.25		1.40	
Maximum Center Distance Variation, in	+0.020		+0.028	
Gear Efficiency	97%		98%	

Summary

Direct Gear Design is an alternative approach to traditional gear design. It is not constrained by predefined tooling parameters and allows analysis of a wide range of parameters for all possible gear combinations in order to find the most suitable solution for a particular custom application. This gear solution can exceed the limits of traditional rack generating methods of gear design.

Direct Gear Design allows reduced stress level compared to traditionally designed gears up to 15 – 30% that can be translated into:

- 15 – 30% increased Load Capacity

- 10 – 20% reduced Size and Weight
- Longer Life
- Cost reduction
- Increased Reliability
- Noise and Vibration reduction (finer pitch, more
  teeth will result higher contact ratio for the given center distance)
- 1 - 2% increased Gear Efficiency (per stage)
- Maintenance Cost reduction
- Other benefits for particular application

Direct gear design for asymmetric tooth profiles opens additional reserves for improvement of gear drives with unidirectional load cycles that are typical for many mechanical transmissions.

Publications about the Direct Gear Design
(could be downloaded from www.akgears.com)

- A. L. Kapelevich, Y. V. Shekhtman, Direct Gear Design: Bending Stress Minimization, Gear Technology, September/October 2003, 44 - 47.
- A. L. Kapelevich, R. E. Kleiss, Direct Gear Design for Spur and Helical Involute Gears, Gear Technology, September/October 2002, 29 - 35.
- A. L. Kapelevich, Geometry and design of involute spur gears with asymmetric teeth, Mechanism and Machine Theory, 35 (2000), 117-130.
- F. L. Litvin, Q. Lian, A. L. Kapelevich, Asymmetric modified gear drives: reduction of noise, localization of contact, simulation of meshing and stress analysis, Computer Methods in Applied Mechanics and Engineering, 188 (2000), 363-390.