High Gear Ratio Epicyclic Drives Analysis

Dr. Alexander Kapelevich

It has been documented that epicyclic gear stages provide high load capacity and compactness to gear drives. This paper will focus on analysis and design of epicyclic gear arrangements that provide extremely high gear ratios. Indeed, a special, two-stage planetary arrangement may utilize a gear ratio of over one hundred thousand to one. This paper presents an analysis of such uncommon gear drive arrangements and defines their major parameters, limitations, and gear ratio maximization approaches. It also demonstrates numerical examples, existing designs, and potential applications.

Introduction

Epicyclic gear stages provide high load capacity and compactness to gear drives. There is a wide variety of different combinations of planetary gear arrangements (Refs. 1–2). For simple, epicyclic planetary stages when the ring gear is stationary, the practical gear ratio range varies from 3:1 to 9:1. For similar epicyclic planetary stages with compound planet gears, the practical gear ratio range varies from 8:1 to 30:1.

This paper presents analysis and design of epicyclic gear arrangements that provide extremely high gear ratios. Using differential-planetary gear arrangements it is possible to achieve gear ratios of several-hundred-to-one in one-stage-drive with common planet gears, and several-thousand-to-one in one-stage drive with compound planet gears. A special two-stage planetary arrangement may utilize a gear ratio of over one-hundred-thousand-to-one.

This paper provides an analysis of such uncommon gear drive arrangements and defines their major parameters, limitations, and gear ratio maximization approaches. It also demonstrates numerical examples, existing designs, and potential applications.

One-Stage Arrangements

There are one-stage differential-planetary arrangements that provide much higher gear ratios. In these arrangements the output shaft is connected to the second rotating ring gear rather than the carrier, as with the epicyclic planetary stages. In this case a carrier does not transmit torque and is called a "cage" because it is simply used to support planet gears.

Figures 1a and 1b present differential-planetary arrangements with compound planet gears. In the arrangement in Figure 1a, the sun gear is engaged with a portion of the planet gear that is in mesh with the stationary ring gear. In this case the gear ratio is:

$$u = \frac{1 + \frac{z_{3a}}{z_1}}{1 - \frac{z_{2b} z_{3a}}{z_{2a} z_{3b}}} \quad (1)$$

where

u = gear ratio
z_1 = sun gear number of teeth
z_{2a} = number of teeth the planet gear engaged with the sun gear and stationary ring gear
z_{2b} = number of teeth the planet gear engaged with the rotating ring gear
z_{3a} = stationary ring gear number of teeth
z_{3b} = rotating ring gear number of teeth

In the arrangement in Figure 1b, the sun gear is engaged with a portion of the planet gear that is in mesh with the rotating ring gear. In this case the gear ratio is:

$$u = \frac{1 + \frac{z_{3a} z_{2b}}{z_1 z_{2a}}}{1 - \frac{z_{3a} z_{2b}}{z_{3b} z_{2a}}} \quad (2)$$

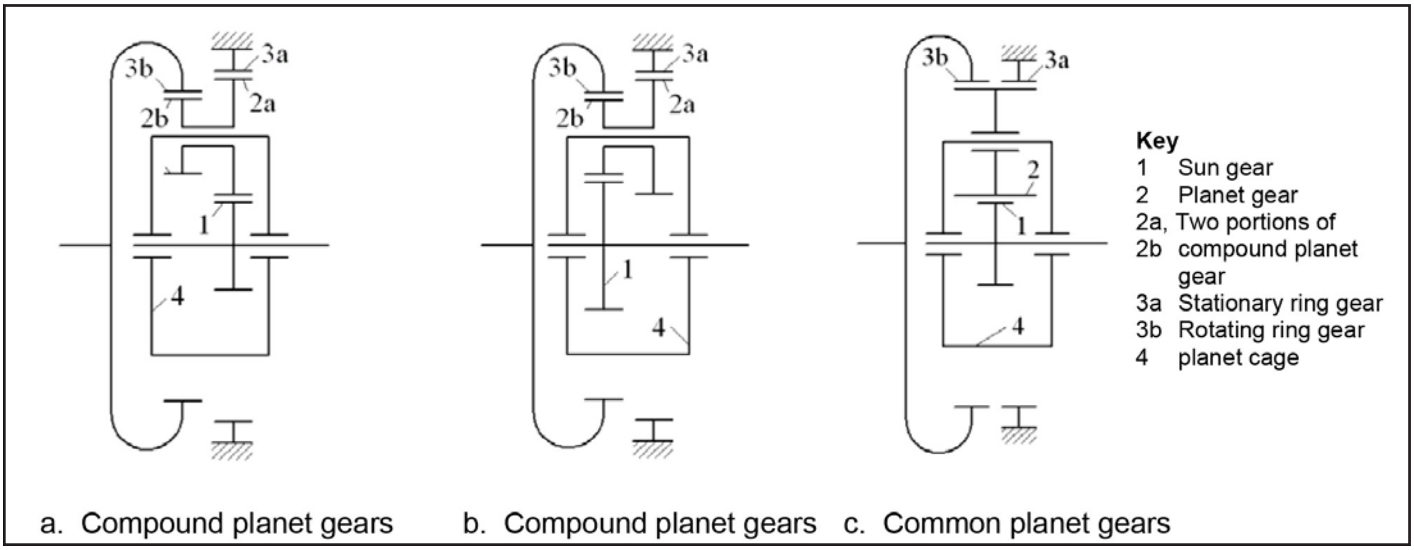

Figure 1 Differential-planetary arrangements.

If a gear ratio is negative, the input and output shaft rotation directions are opposite.

All gear meshes in differential-planetary arrangements have the same center distance. This condition allows for definition of relations between the operating modules, m_w, or diametral pitches, DP_w. For the arrangement in Figure 1a they are:

$$m_{w12a}(z_1+z_{2a})=m_{w2a3a}(z_{3a}-z_{2a})=m_{w2b3b}(z_{3b}-z_{2b}) \tag{3}$$

or

$$\frac{z_1+z_{2a}}{DP_{w12a}}=\frac{z_{3a}-z_{2a}}{DP_{w2a3a}}=\frac{z_{3b}-z_{2b}}{DP_{w2b3b}} \tag{4}$$

The relationship between operating pressure angles in the gear meshes z_1-z_{2a} and $z_{2a}-z_{3a}$ is defined by Equation 5 as:

$$\frac{\cos\alpha_{w2a-3a}}{\cos\alpha_{w1-2a}}=\frac{z_1+z_{2a}}{z_{3a}-z_{2a}} \tag{5}$$

where

α_{w1-2a} operating pressure angle in a mesh of the sun gear and the planet gear engaged with the stationary ring gear

α_{w2a-3a} operating pressure angle in the planet/stationary ring gear mesh

Similar to the arrangement in Figure 1b:

$$m_{w12b}(z_1+z_{2b})=m_{w2b3b}(z_{3b}-z_{2b})=m_{w2a3a}(z_{3a}-z_{2a}) \tag{6}$$

or

$$\frac{z_1+z_{2b}}{DP_{w12b}}=\frac{z_{3b}-z_{2b}}{DP_{w2b3b}}=\frac{z_{3a}-z_{2a}}{DP_{w2a3a}} \tag{7}$$

The relationship between operating pressure angles in the gear meshes z_1-z_{2b} and $z_{2b}-z_{3b}$ is defined by equation:

$$\frac{\cos\alpha_{w2b-3b}}{\cos\alpha_{w1-2b}}=\frac{z_1+z_{2b}}{z_{3a}-z_{2b}} \tag{8}$$

where

α_{w1-2b} operating pressure angle in a mesh of the sun gear and the planet gear engaged with the rotating ring gear

α_{w2b-3b} operating pressure angle in the planet/rotating ring gear mesh

In differential-planetary arrangements with compound planet gears, operating pressure angles in the planet/stationary ring gear mesh and in the planet the planet/rotating ring gear mesh can be selected independently. This allows for balancing specific sliding velocities in these meshes to maximize gear efficiency, which could be 80–90% — depending on the gear ratio (Ref. 2). The maximum gear ratio in such arrangements is limited by possible tip/tip interference of the neighboring planet gears. In order to avoid this interference the following condition should be satisfied:

For the arrangement in Figure 1a:

$$z_{2a}=\frac{z_1\sin\left(\frac{\pi}{n_w}\right)-2h_{2a}}{1-\sin\left(\frac{\pi}{n_w}\right)} \tag{9}$$

For the arrangement in Figure 1b:

$$z_{2b}=\frac{z_1\sin\left(\frac{\pi}{n_w}\right)-2h_{2b}}{1-\sin\left(\frac{\pi}{n_w}\right)} \tag{10}$$

where

n_w number of planets

h_{2a}, h_{2b} operating addendum coefficients of the planet gears z_{2a} and z_{2b}, accordingly.

Maximum gear ratio values for the differential-planetary arrangement with the compound planet gears (assuming $h_{2a}=h_{2b}=1.0$) are shown in Table 1.

The assembly condition for these gear arrangements is:

$$\frac{z_{3a}-z_{3b}}{n_w}=\text{integer} \tag{11}$$

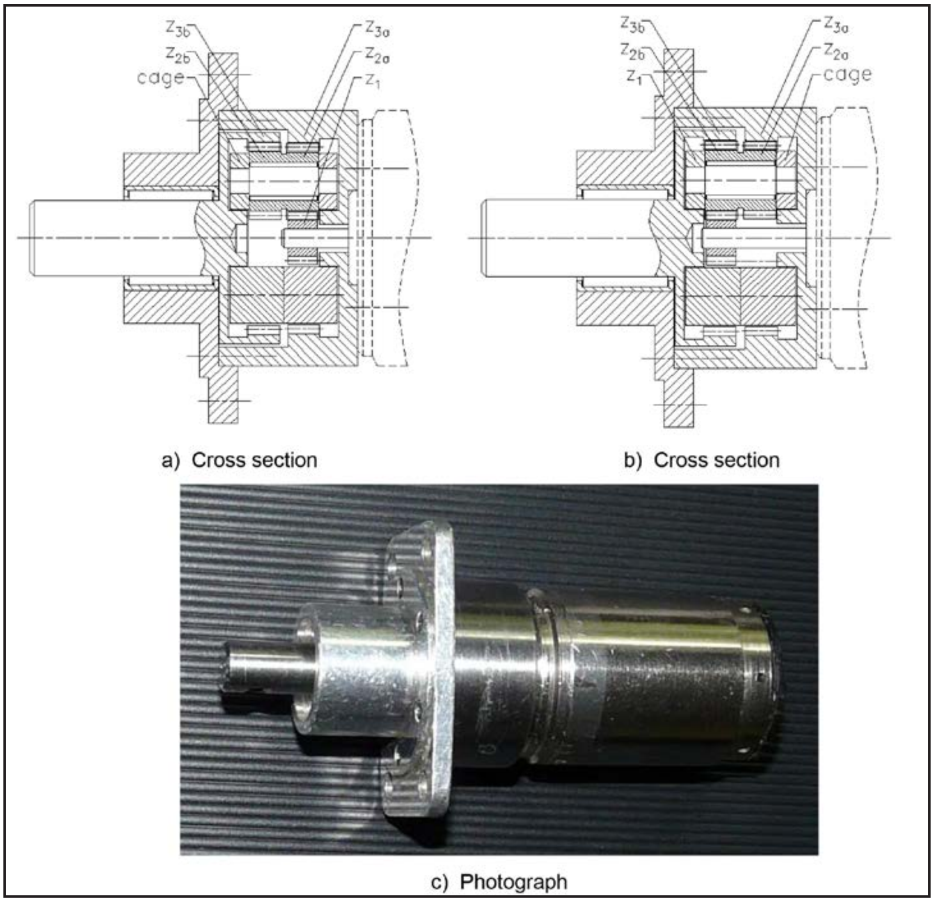

Figure 2 **Differential-planetary gear actuators with compound planet gears.**

Two parts of a compound planet gear should be angularly aligned for proper assembly. This is typically achieved by aligning the axes of one tooth of each part of the compound planet gear, which makes its fabrication more complicated. Assembly of such gear drives requires certain angular positioning of planet gears. All these factors increase the cost of this type of gear drive. Examples of differential-planetary gear actuators with compound planet gears are shown in Figure 2.

Table 1 **Maximum gear ratio values for differential-planetary arrangements with compound planet gears**

Number of planets	Sun gear tooth number	Maximum gear ratio*
3	10	±1579:1
	15	±2857:1
	25	±5183:1
4	10	±144:1
	15	±273:1
	25	±518:1
5	10	±49:1
	15	±80:1
	25	±162:1

* Sign "+" if the input and output shaft rotation directions are the same, sign"-" if they are opposite.

A simplified version of the one-stage, differential-planetary arrangement is shown in Figure 1c; this arrangement does not use the compound planet gear. The common planet gear is engaged with the sun gear, and both the stationary and the rotating ring gears. This does not allow for specific sliding velocities in each mesh to be equalized, resulting in a reduction of gear efficiency of about 70–84% (Ref. 2). However, the assembly of such gear drives does not require certain angular positioning of planet gears, and their manufacturing cost is significantly lower for drives with the compound planet gears. An example of the differential-planetary gear actuator with common planet gears is shown in Figure 3.

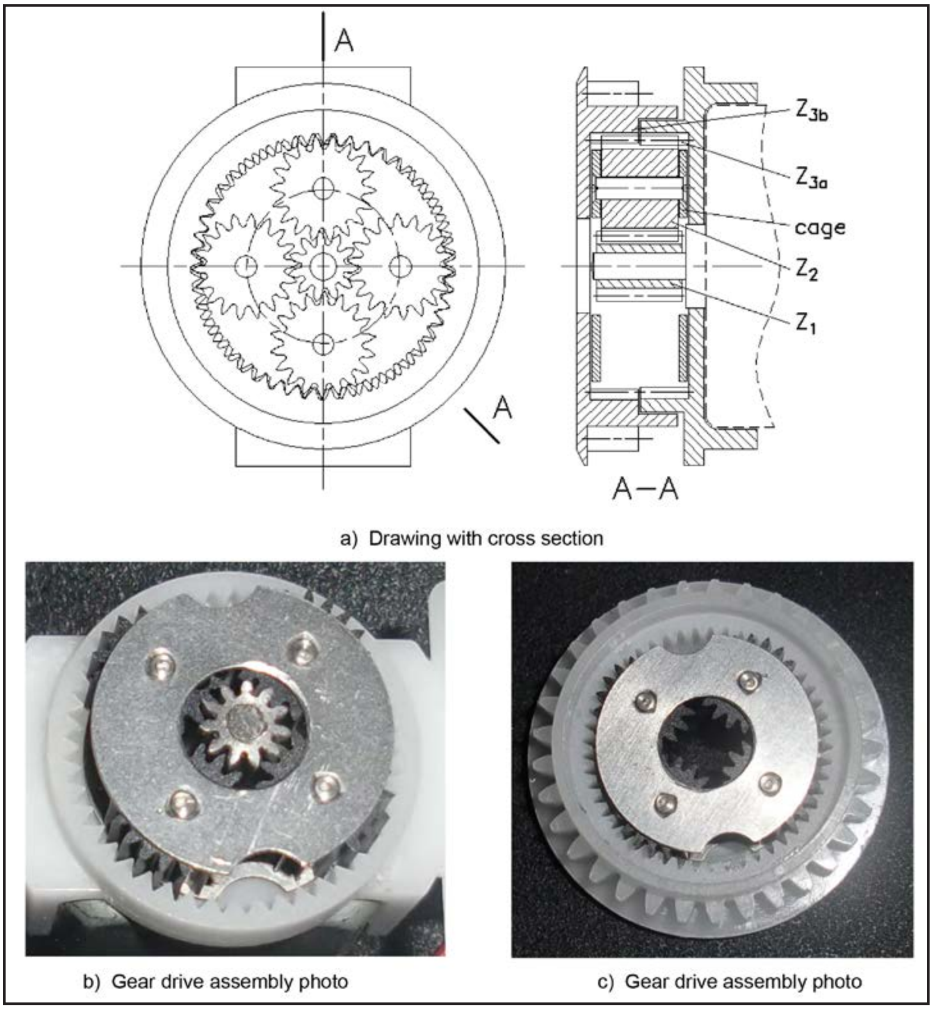

Figure 3 Differential-planetary gear actuator.

Relations between operating pressure angles in the gear meshes are defined by Equations 12–15:

$$\frac{\cos \alpha_{w2\text{-}3a}}{\cos \alpha_{w1\text{-}2}} = \frac{z_1 + z_2}{z_{3a} - z_2} \tag{12}$$

$$\frac{\cos \alpha_{w2\text{-}3b}}{\cos \alpha_{w1\text{-}2}} = \frac{z_1 + z_2}{z_{3b} - z_2} \tag{13}$$

$$\frac{\cos \alpha_{w2\text{-}3b}}{\cos \alpha_{w2\text{-}3a}} = \frac{z_{3a} + z_2}{z_{3b} - z_2} \tag{14}$$

where

$\alpha_{w1\text{-}2}$ operating pressure angle in sun/planet gear mesh
$\alpha_{w2\text{-}3a}$ operating pressure angle in planet/stationary ring gear mesh
$\alpha_{w2\text{-}3b}$ operating pressure angle in planet/rotating ring gear mesh

A gear ratio is

$$u = \frac{1 + \frac{z_{3a}}{z_1}}{1 - \frac{z_{3a}}{z_{3b}}} \tag{15}$$

Maximum gear ratio values for the differential-planetary arrangement with the common planet gears (assuming $h_{2a} = h_{2b} = 1.0$) are shown in the Table 2.

In differential-planetary arrangements (Fig. 1), tangent forces applied to the planet gear teeth from the stationary and rotating ring gears are unbalanced,

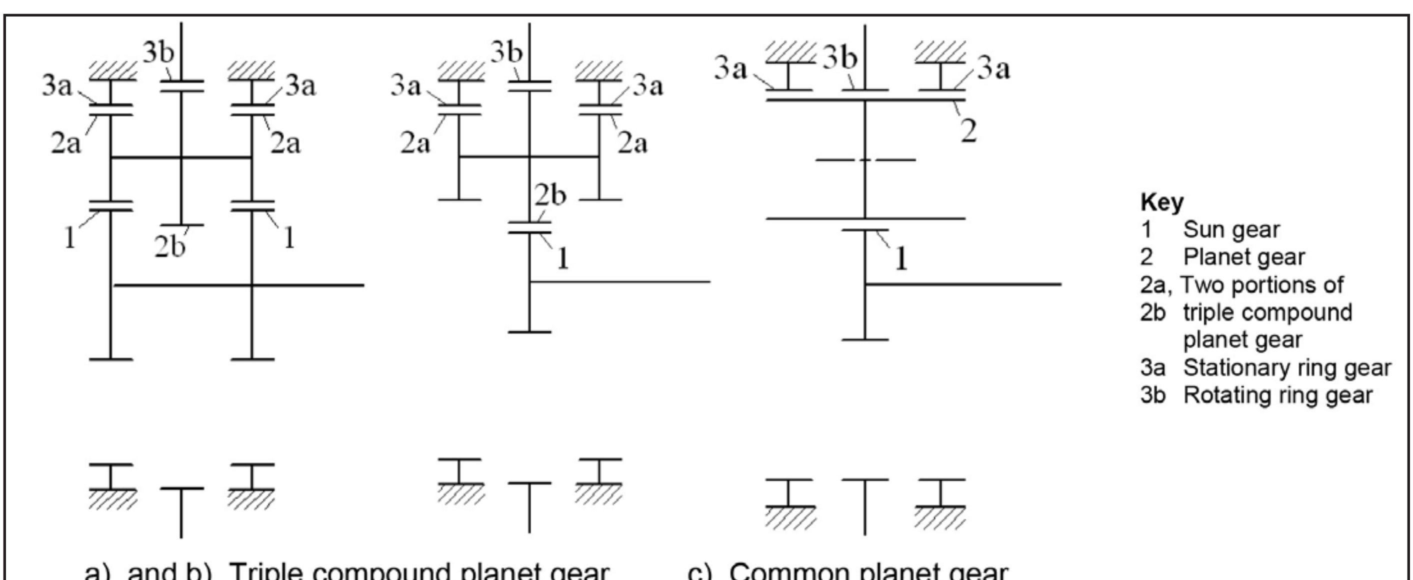

Figure 4 Differential-planetary arrangements without planet gear cage.

as they lie on different parallel planes and have opposite directions. A sturdy planet cage is required to avoid severe planet gear mesh misalignment. There are some gear drives that use the differential-planetary arrangements with balanced planet gear tangent forces (Fig. 4). In this case, the triple-compound planet gears (Figs. 4a and 4b) are used. They have identical gear profiles on their end portions that are engaged with the two identical stationary ring gears. The middle portion of such planet gears has a different profile than those on the ends, and is engaged with the rotating ring gear. The arrangement in Figure 4c has common planet gears engaged with the sun gear, two stationary ring gears, and one rotating ring gear. These types of differential-planetary drives typically do not have the cage and bearings, because the planet gear forces are balanced and planet gears themselves work like the roll bearings for radial support of the rotating ring gear.

Two-Stage Arrangements

In most conventional, two-stage, epicyclic arrangements, the gear ratio usually does not exceed 100:1 — although there are possible arrangements that allow a significant increase in the gear ratio (Ref. 3). Figure 5 shows the planetary gear arrangement "A" with the sun gears of the first and second stages connected together and the compound cage supporting the planet gears of both the first and second stages.

A sketch of the gearbox with arrangement A is presented in Figure 6. Both sun gears are connected to the input shaft and are engaged with the planet gears of the first and second stages, respectively. The first-stage ring gear is stationary and is

Table 2 Maximum gear ratio values for differential-planetary arrangements with compound planet gears		
Number of planets	Sun gear tooth number	Maximum gear ratio
3	10	±405:1
	15	±767:1
	25	±1432:1
4	10	±59:1
	15	±101:1
	25	±198:1
5	10	±20:1
	15	±32:1
	25	±70:1

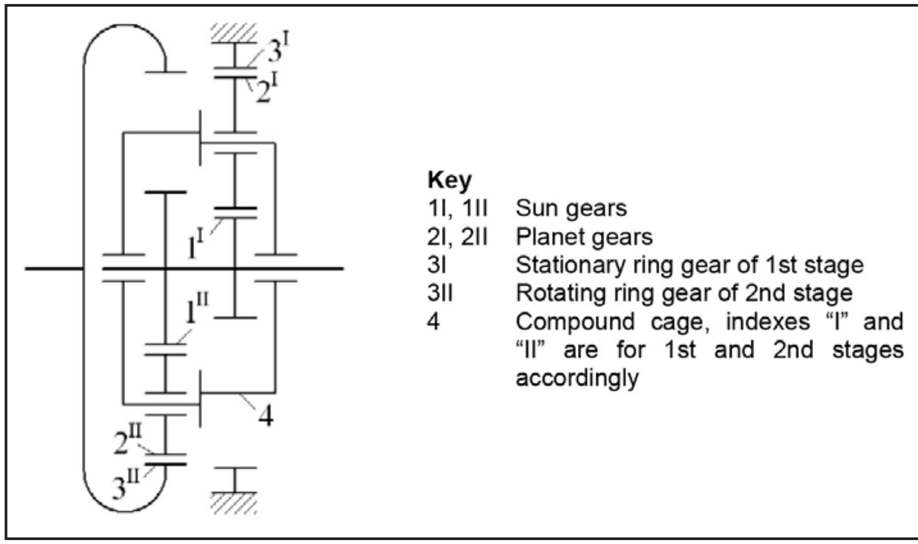

Figure 5 Two-stage planetary (arrangement A) with connected sun gears of first and second stages and compound cage.

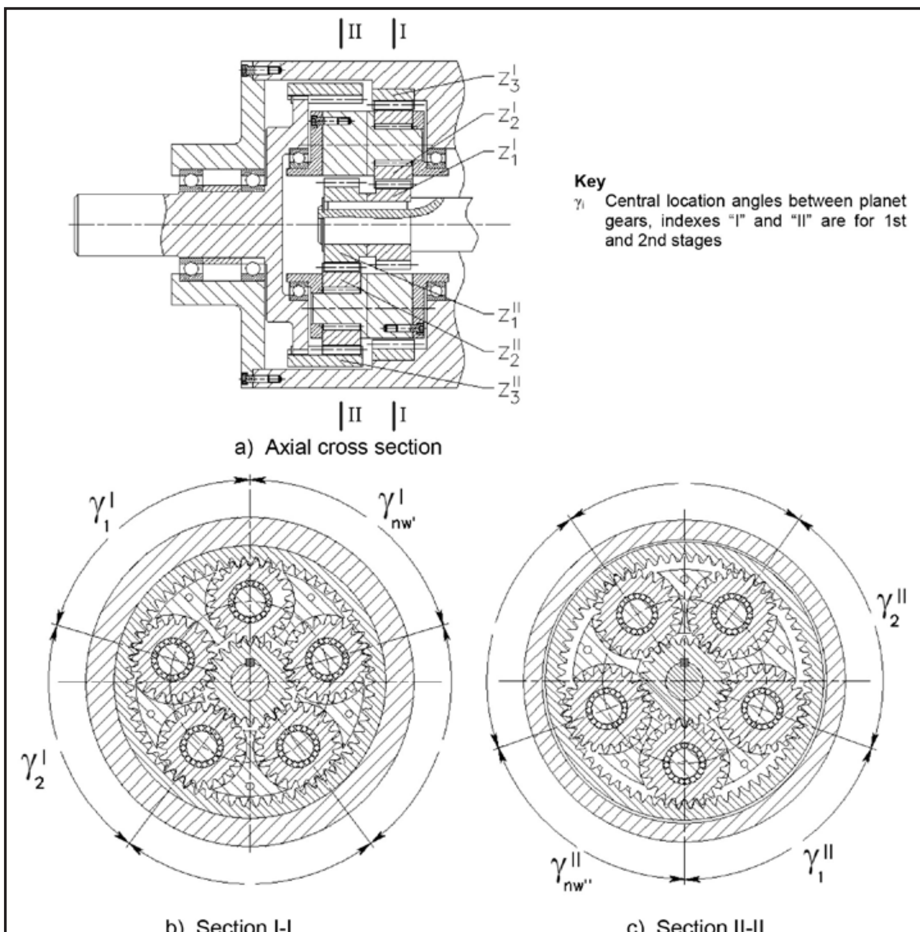

Figure 6 Two-stage planetary gearbox (arrangement A) with connected sun gears of first and second stages and compound cage.

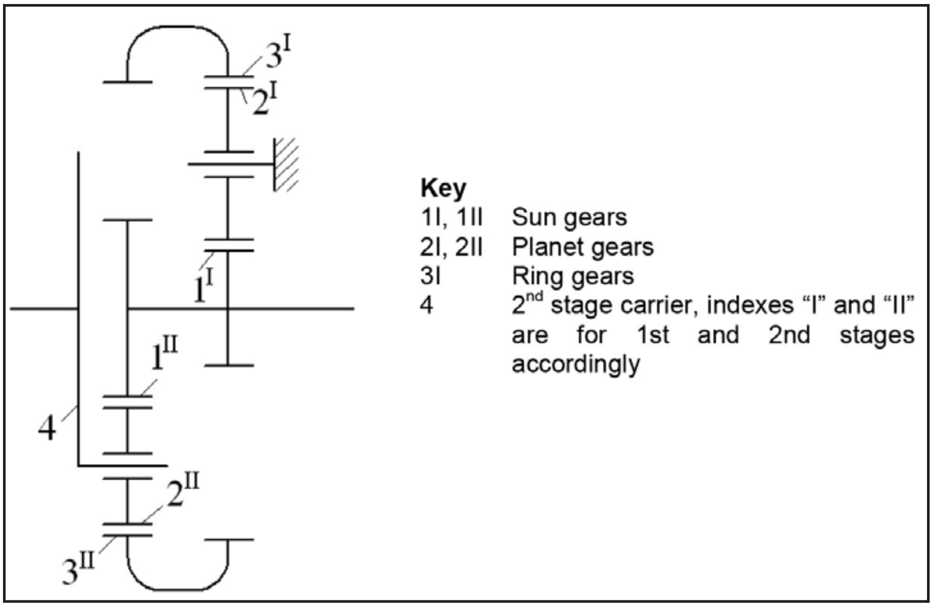

Figure 7 **Two-stage planetary (arrangement B) with sun gears and ring gears of first and second stages joined.**

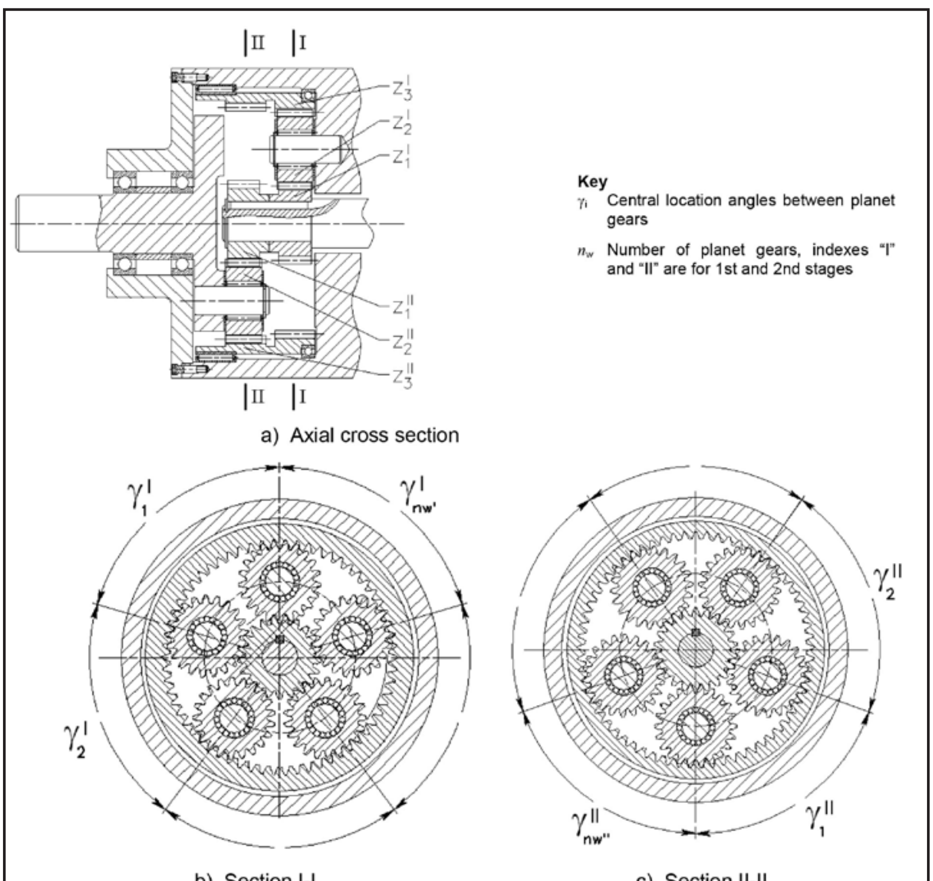

Figure 8 **Two-stage planetary gearbox (arrangement B) with sun gears and ring gears of 1st and 2nd stages joined.**

connected with the gearbox housing. It is engaged with the first-stage planet gears. The compound cage practically contains the first- and second-stage cages joined together. The ring gear of the second stage is engaged with the second-stage planet gears and connected to the output shaft. The gear ratio of arrangement A is:

$$u = \frac{z_3^{II}\left(z_1^{I} + z_3^{I}\right)}{z_1^{I} z_3^{II} - z_1^{II} z_3^{I}} \quad (16)$$

where

z_1^I, z_1^{II} = numbers of teeth of the sun gears of the 1st and 2nd stages

z_2^I, z_2^{II} = numbers of teeth of the planet gears of the 1st and 2nd stages

z_3^I, z_3^{II} = numbers of teeth of the ring gears of the 1st and 2nd stages

Figure 7 shows the alternative gear arrangement B with the sun gears of both stages joined together and the ring gears of both stages also connected.

A sketch of the gearbox with the alternative arrangement B is presented in Figure 8.

Both sun gears are connected to the input shaft and engaged with the planet gears of the first and second stages, respectively. The shafts supporting the first-stage planet gears are connected (pressed in, for example) to the gearbox housing. Both ring gears are joined and engaged with the planet gears of the first and second stages; the second-stage carrier is joined with the output shaft. The gear ratio of arrangement B is:

$$u = \frac{z_3^{I}\left(z_1^{II} + z_3^{II}\right)}{z_1^{II} z_3^{I} - z_1^{I} z_3^{II}} \quad (17)$$

The maximum gear ratios of these two-stage planetary arrangements A and B are achieved when the denominator of Equations 16 and 17 is equal to 1 or –1. This condition can be presented as:

$$\left| z_1^{I} z_3^{II} - z_1^{II} z_3^{I} \right| = 1 \quad (18)$$

When this denominator is 1, the input and output shafts are rotating in the same direction. When it is less –1, the input and output shafts are rotating in opposite directions. If a number of planet gears are more than 1 (n_w^I>1 and n_w^{II}>1), Equation 18 requires irregular, angular positioning of the planet gears in one or both planetary stages. This means that the central location angles γ_i between planet gears in one or both stages are not identical (see Figs. 6b, 6c, 8b and 8c). A definition of

Table 3 Maximum achievable gear ratios depending on numbers of teeth

Ring gear number of teeth, z_3^I and z_3^{II}	Maximum gear ratio
100	±14,000:1
200	±66,000:1
300	±160,000:1
400	±280,000:1

Table 4 Gear ratio example for planetary arrangements A and B

Arrangement		A (Figure 5)	B (Figure 7)
1st stage	Sun gear number of teeth	21	21
	Planet gear number of teeth	21	21
	Ring gear number of teeth	62	62
	Number of planet gears	5	5
2nd stage	Sun gear number of teeth	22	22
	Planet gear number of teeth	22	22
	Ring gear number of teeth	65	65
	Number of planet gears	5	5
Gear ratio		5395:1	-5394:1

the central angles with irregular angular positioning of the planet gears that provide proper assembly is described in Reference 3.

The neighboring planet gears located at the minimum central angles must be checked for the possibility of tip/tip interference. Irregular angular positioning of the planet gears may result in an imbalance in the planetary stage. This must be avoided by carrier assembly balancing.

Application of the two-stage planetary arrangements A and B allows very high gear ratio values to be achieved. In practice, these values are limited only by the number of teeth of ring gears z_3^I and z_3^{II}. Table 3 presents maximum achievable gear ratios depending on the number of teeth of ring gears z_3^I and z_3^{II}.

Unlike conventional, two-stage epicyclic arrangements, in the planetary arrangements A and B a total gear ratio does not depend on internal gear ratios in each stage; this allows increasing the number of planet gears. An example of the gear ratio for the planetary arrangements A and B is shown in Table 4.

The efficiency of these two-stage planetary gear arrangements is in opposite proportion to gear ratio, and is much lower than for conventional, two-stage, epicyclic gear arrangements. One potential area of application is in different positioning systems that need very low-output RPM and typically do not require high-output torque.

Potential applications

Potential areas of application of high gear ratio, one-stage differential-planetary arrangements include different aerospace drives, such as flap actuators, robotic mechanisms, etc.

Extremely high gear ratio, two-stage planetary arrangements can be applied in different positioning systems that need very low-output RPM and typically do not require very high-output torque, such as the tracking system gear drives of solar batteries or the mirrors of solar power stations.

Summary

High gear ratio, one-stage differential-planetary arrangements with compound and common planet gears are described. Gear ratio equations and maximum values are defined.

Extremely high gear ratio, two-stage planetary arrangements are described. Gear ratio equations and maximum achievable values are defined.

Potential applications of high gear ratio, one-stage and two-stage planetary drives are suggested.

References

1. Henriot, G. "Gears and Planetary Geartrains," Brevini, Reggio nell'Emilia, 2001.
2. Kudriavtsev, V.N. "Planetary Gears," *Mashinostroenie*, Leningrad, 1966.
3. Kapelevich, A.L. *Direct Gear Design,* CRC Press, 2013.

Alex Kapelevich possesses more than 30 years of custom gear research and design experience, as well as over 100 successfully accomplished projects for a variety of gear applications. His company, AKGears, provides consulting services—from complete gear train design (for customers without sufficient gear expertise) to retouching (typically tooth and fillet profile optimization) of existing customers' designs—in the following specific areas: traditional or direct gear design; current design refinement; R&D; failure and testing analysis. The company provides gear drive design optimization for increased load capacity; size and weight reduction; noise and vibration reduction; higher gear efficiency; backlash minimization; increased lifetime; higher reliability; cost reduction; and gear ratio modification and adjustment. Kapelevich is the author of numerous technical publications and patents, and is a member of the AGMA Aerospace and Plastic Gearing Committees, SME, ASME and SAE International. He holds a Ph.D. in mechanical engineering from Moscow State Technical University and a Master Degree in mechanical engineering from the Moscow Aviation Institute.